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The stochastic heat equation: Feynman-Kac formula and intermittence. (English)

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Summary: We study, in one space dimension, the heat equation with a random potential that is a white noise in space and time. This equation is a linearized model for the evolution of a scalar field in a space-time-dependent random medium. It has also been related to the distribution of two-dimensional directed polymers in a random environment, to the KPZ model of growing interfaces, and to the Burgers equation with conservative noise. We show how the solution can be expressed via a generalized Feynman-Kac formula. We then investigate the statistical properties: the two-point correlation function is explicitly computed and the intermittence of the solution is proven. This analysis is carried out showing how the statistical moments can be expressed through local times of independent Brownian motions.

MSC:

- 60H15 Stochastic partial differential equations (aspects of stochastic analysis)
- 60J50 Boundary theory for Markov processes
- 82C31 Stochastic methods (Fokker-Planck, Langevin, etc.) applied to problems in time-dependent statistical mechanics

Cited in **2** Reviews
Cited in **78** Documents

Keywords:

stochastic partial differential equations; random media; moment Lyapunov exponents; local times

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