Mikula, Karol; **Sevcovic, Daniel**

Evolution of plane curves driven by a nonlinear function of curvature and anisotropy. (English) Zbl 0980.35078

SIAM J. Appl. Math. 61, No. 5, 1473-1501 (2001).

The authors consider a family of evolving curves whose position vector $\vec{X}$ satisfies the equation $\partial_t \vec{X} = \beta(k,\nu)\vec{N} + \alpha\vec{T}$. While the normal component of the velocity, which is a function of the curvature k and the tangential angle ν, is the only one that determines the shape of the evolving curves, the tangential velocity α proves to be useful in numerical schemes. It is shown that if α is the solution of a certain nonlocal equation, which has the effect of uniformly redistributing the material points along the curve, the numerical scheme derived via the flowing finite volumes prevents the formation of computational instabilities.

The first part of the paper is devoted to the the study of the short time existence of classical solutions, first in the case when $\partial_k\beta$ is nondegenerate in the sense that the derivative of (class C^2) β with respect to k is uniformly bounded below and above by two strictly positive constants, and then in the singular case when $\beta(k,\nu) = \gamma(\nu)|k|^{m-1}k$ for $m \in (0,1) \cup (1,2]$, extending the result due to Angenent, Sapiro and Tannenbaum for $m = 1/3$. The second part deals with the numerical solutions. In particular, it presents a comparison between results obtained by considering trivial and, respectively, nontrivial tangential velocities.

Reviewer: Alina Stancu (Villeneuve D'Ascq)

MSC:

35K65	Degenerate parabolic equations
65N40	Method of lines for boundary value problems involving PDEs
65N30	Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
53C44	Geometric evolution equations (mean curvature flow, Ricci flow, etc.) (MSC2010)

Keywords:

tangential velocity; flowing finite volumes; computational instabilities

Full Text: DOI