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**Normal automorphisms of a free pro- $p$ -group in the variety  $\mathcal{N}_2\mathcal{A}$ .** (English. Russian original)

Zbl 0976.20017

Algebra Logika 35, No. 3, 249-267 (1996); translation in Algebra Logic 35, No. 3, 139-148 (1996).

Summary: An automorphism of a (profinite) group is called normal if each (closed) normal subgroup is left invariant by it. An automorphism of an abstract group is  $p$ -normal if each normal subgroup of  $p$ -power index, where  $p$  is a prime, is left invariant. Obviously, an inner automorphism of a group is normal and  $p$ -normal. For some groups, the converse was stated to be likewise true. N. Romanovskij and V. Boluts, for instance, established that for free solvable pro- $p$ -groups of derived length 2, there exist normal automorphisms that are not inner. Let  $\mathcal{N}_2$  be the variety of nilpotent groups of class 2 and  $\mathcal{A}$  the variety of Abelian groups. We prove the following results: (1) If  $p$  is a prime number distinct from 2, then a normal automorphism of a free pro- $p$ -group of rank  $\geq 2$  in  $\mathcal{N}_2\mathcal{A}$  is inner (Theorem 1): (2) if  $p$  is a prime number distinct from 2, then a  $p$ -normal automorphism of an abstract free  $\mathcal{N}_2\mathcal{A}$ -group of rank  $\geq 2$  is inner (Theorem 2).

**MSC:**

- 20E18 Limits, profinite groups
- 20E36 Automorphisms of infinite groups
- 20E10 Quasivarieties and varieties of groups
- 20E28 Maximal subgroups

**Keywords:**

normal automorphisms; inner automorphisms; free solvable pro- $p$ -groups; varieties of nilpotent groups;  $p$ -normal automorphisms

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