

Azagra, D.; Deville, R.

James' theorem fails for starlike bodies. (English) Zbl 0983.46016
J. Funct. Anal. 180, No. 2, 328-346 (2001).

A closed subset A of a Banach space is said to be a starlike body provided A has a non-empty interior $\text{int } A$ and there exists a point $x_0 \in \text{int } A$ such that each ray emanating from x_0 meets the boundary of A at most once. Since every convex body is a starlike body, one may ask whether the famous James' theorem on characterization of reflexivity remains true when one replaces the word "convex" with the word "starlike" in this theorem. The authors disprove this conjecture by showing that in the Hilbert space ℓ_2 there exist a C^∞ smooth bounded starlike body A and a one-codimensional subspace $H \subseteq \ell_2$ with the property that for no $y \in \partial A$ is the hyperplane $y + H$ tangent to A at y .

Reviewer: L.Janos (Kent/Ohio)

MSC:

46B20 Geometry and structure of normed linear spaces
46G20 Infinite-dimensional holomorphy

Cited in **3** Reviews
Cited in **10** Documents

Keywords:

starlike body; convex body; James' theorem; characterization of reflexivity

Full Text: [DOI](#)

References:

- [1] Azagra, D., Diffeomorphisms between spheres and hyperplanes in infinite-dimensional Banach spaces, *Studia math.*, 125, 179-186, (1997) · [Zbl 0899.58007](#)
- [2] Azagra, D.; Dobrowolski, T., Smooth negligibility of compact sets in infinite-dimensional Banach spaces, with applications, *Math. ann.*, 312, 445-463, (1998) · [Zbl 0917.46007](#)
- [3] Azagra, D.; Gómez, J.; Jaramillo, J.A., Rolle's theorem and negligibility of points in infinite-dimensional Banach spaces, *J. math. anal. appl.*, 213, 487-495, (1997) · [Zbl 0897.46027](#)
- [4] D. Azagra, and, M. Jiménez-Sevilla, Rolle's theorem is either false or trivial in infinite-dimensional Banach spaces, preprint, 2000.
- [5] Azagra, D., Smooth negligibility and subdifferential calculus in Banach spaces, with applications, (1997), Universidad Complutense de Madrid
- [6] Bates, S.M., On smooth, nonlinear surjections of Banach spaces, *Israel J. math.*, 100, 209-220, (1997) · [Zbl 0898.46044](#)
- [7] Cartan, H., *Calcul différentiel*, (1967), Hermann Paris
- [8] Debs, G.; Godefroy, G.; Saint Raymond, J., Topological properties of the set of norm-attaining linear functions, *Canad. J. math.*, 47, 318-329, (1995) · [Zbl 0831.46009](#)
- [9] Deville, R.; Godefroy, G.; Zizler, V., Smoothness and renormings in Banach spaces, *Pitman monographs and surveys in pure and applied mathematics*, 64, (1993), Pitman London
- [10] Hájek, P., Smooth functions on c_0 , *Israel J. math.*, 104, 17-27, (1998)
- [11] James, R.C., Weakly compact sets, *Trans. amer. math. soc.*, 113, 129-140, (1964) · [Zbl 0129.07901](#)
- [12] Jiménez-Sevilla, M.; Moreno, J.P., A note on norm attaining functions, *Proc. amer. math. soc.*, 126, 1989-1997, (1998) · [Zbl 0894.46011](#)
- [13] Leach, E.B.; Whitfield, J.H.M., Differentiable functions and rough norms on Banach spaces, *Proc. amer. math. soc.*, 120-126, (1972) · [Zbl 0236.46051](#)
- [14] Shkarin, S.A., On Rolle's theorem in infinite-dimensional Banach spaces, *Mat. Z.*, 51, 128-136, (1992) · [Zbl 0786.46044](#)

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