Letac, Gérard

Lectures on natural exponential families and their variance functions. (English)

Zbl 0983.62501
Monografías de Matemática (Rio de Janeiro). 50. Rio de Janeiro: Instituto de Matemática Pura e Aplicada (IMPA). 128 p. (1992).

From a paper of *R.G. Laha* and *E. Lukacs* [Biometrika 47, 335-343 (1960; Zbl 0093.16002)] it follows that if $X_1, \dots, X_n$ ($n \geq 2$) are independent, identically distributed square integrable nondegenerate random variables, then, for some constants a, b, c, we have

$$E(X_1^2 - aX_1X_2 - bX_1 \mid \sum_{i=1}^{n} X_i) = c$$

almost surely if and only if some linear function of X_1 (and hence of each X_i) is either binomial or negative binomial or Poisson or normal or gamma or Meixner hypergeometric. [For details of Meixner hypergeometric variables, see the paper of *C.D. Lai*, Aust. J. Stat. 24, 221-233 (1982; Zbl 0492.62013)]. Also, it is clear from the cited reference of Laha and Lukacs, that in this characterization the distributions are classified according to the values of a, b and c. (Incidentally, Laha and Lukacs gave the result in a more general form. However, that the general result is a corollary to the above specialized result is not difficult to see.) Subsequently, *C.N. Morris* [Ann. Stat. 10, 65-80 (1982; Zbl 0498.62015)] identified natural exponential families that have variances equal to quadratic functions of the corresponding means. However, the identification in question was shown implicitly to be a corollary to the result of Laha and Lukacs, referred to above, by the reviewer [Teor. Veroyatn. Primenen. 24, 424-427 (1979; Zbl 0436.60017)]. It is unfortunate that *C.N. Morris* [op. cit.; Ann. Stat. 11, 515-529 (1983; Zbl 0521.62014)] and several other subsequent authors have missed this reference. It is also of relevance to observe here that a result of *D.V. Gokhale* [J. Ind. Stat. Assoc. 18, 81-84 (1980)] is a corollary to the 1982 result of Morris that we have mentioned.

This nicely written monograph on natural exponential families and their variance functions has its main theme in Morris' 1982 result. It consists of six chapters with the following titles: The class $\mathcal{M}(\mathbf{R}^n)$ (i.e., the class of positive Radon measures on $\mathbf{R}^n$); Natural exponential families and their variance functions; Morris classes of natural exponential families; Bayesian theory and natural exponential families; The Babel class of NEFs (i.e., natural exponential families); and Analogues of Morris families in several dimensions, respectively. In Chapter 1, some interesting properties of Laplace transforms, infinitely divisible distributions, etc., are discussed, and in the remainder of the monograph an account of the recent developments on variance functions of natural exponential families together with some basic properties is given. Amongst the results that are presented, there are several that deal with aspects of Morris' result or its generalizations. Some related questions in Bayesian inference and those on properties of the Wishart distribution are also addressed.

This study provides one with a good picture of the current state of the literature in the area with which it is concerned, and the reviewer believes that anyone interested in structural aspects of probability distributions would benefit from it. [Proposition 6.6 of Chapter 1 also appeared in a paper by the reviewer and *M. Sreehari*, Z. Wahrscheinlichkeitstheor. Verw. Geb., 38, 217-222 (1977; Zbl 0353.60025)].

Reviewer: D.N.Shanbhag (MR 94f:60002)

MSC:

62E10	Characterization and structure theory of statistical distributions
62-02	Research exposition (monographs, survey articles) pertaining to statistics
60E05	Probability distributions: general theory
60-02	Research exposition (monographs, survey articles) pertaining to probability theory