Barcucci, E.; **Del Lungo, A.**; **Frosini, A.**; **Rinaldi, S.**

A technology for reverse-engineering a combinatorial problem from a rational generating function. (English) Zbl 0984.05005

Adv. Appl. Math. 26, No. 2, 129-153 (2001).

In [*N. Chomsky* and *M. P. Schützenberger*, The algebraic theory of context-free languages, Comput. Program. Formal Syst., 118-161 (1963; Zbl 0148.00804)] a methodology is proposed for determining the generating function of an unambiguous context-free language L from an unambiguous grammar that generates L. The article under review considers the reverse process.

Authors' abstract: We tackle the problem of giving, by means of a regular language, a combinatorial interpretation of a positive sequence (f_n) defined by a linear recurrence with integer coefficients. We propose two algorithms able to determine if the rational generating function of (f_n), $f(x)$, is the generating function of some regular language, and, in the affirmative case, to find it. We illustrate some applications of this method to combinatorial object enumeration problems and bijective combinatorics and discuss an open problem regarding languages having a rational generating function.

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MSC:

05A15 Exact enumeration problems, generating functions
68Q45 Formal languages and automata
68R05 Combinatorics in computer science
05B50 Polyominoes

Keywords:

generating function; context-free language; regular language; combinatorial interpretation; linear recurrence; enumeration

Software:

OEIS

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