Rhin, Georges; **Viola, Carlo**
The group structure for $\zeta(3)$. (English) Zbl 1004.11042
Acta Arith. 97, No. 3, 269-293 (2001).

R. Apéry [Astérisque 61, 11-13 (1979; Zbl 0401.10049)] proved the irrationality of $\zeta(3)$ and gave the irrationality measure $\mu(\zeta(3)) < 13.41782\dots$. The authors [Acta Arith. 77, 23-56 (1996; Zbl 0864.11037)] obtained the record irrationality measure for $\zeta(2)$, viz. $\mu(\zeta(2)) < 5.441243\dots$ by an arithmetical study of a family of double integrals lying in $\mathbb{Q} + \mathbb{Z}\zeta(2)$. In the present paper, the authors succeed in adapting their method to a family of triple integrals lying in $\mathbb{Q} + \mathbb{Z}\zeta(3)$ and obtain $\mu(\zeta(3)) < 5.513891\dots$.

The triple integrals are given by

$$\int_0^1\int_0^1\int_0^1 \frac{x^h(1-x)^l y^k(1-y)^s z^j(1-z)^q}{(1-(1-xy)z)^{q+h-r}}\frac{dxdydz}{1-(1-xy)z}.$$

The birational transformation

$$X = (1-y)z, \qquad Y = \frac{(1-x)(1-z)}{1-(1-xy)z}, \qquad Z = \frac{y}{1-(1-y)z}$$

of period 8 produces a cyclic permutation of the 8 parameters in the triple integral and provides the basis of the algebraic structure at the heart of the proof.

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MSC:

11J82 Measures of irrationality and of transcendence
11M06 $\zeta(s)$ and $L(s,\chi)$

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