Azagra, Daniel; Jiménez-Sevilla, Mar

The failure of Rolle's theorem in infinite-dimensional Banach spaces. (English) Zbl 0995.46025

J. Funct. Anal. 182, No. 1, 207-226 (2001).

A bump is nonzero real function with bounded support. The authors prove that if a Banach space X admits a C^p smooth (Lipschitz) bump then it admits another C^p smooth (Lipschitz) bump $f : X \to [0, 1]$ with the property that $f'(x) \neq 0$ for all x in the interior of the support of f. This is applied to discussing Rolle's theorem, deleting diffeomorphisms, and Brouwer fixed points in infinite dimensions.

Reviewer: S.S.Kutateladze (Novosibirsk)

MSC:

46G05 Derivatives of functions in infinite-dimensional spaces
47H10 Fixed-point theorems

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