Hille, Lutz; **de la Peña, José Antonio**
Distinguished slopes for quiver representations. (English) Zbl 1011.16012
Bol. Soc. Mat. Mex., III. Ser. 7, No. 1, 73-83 (2001).

Let Q be finite connected quiver without oriented cycles. A slope $\mu(M) = \theta(M)/k(M)$ is a function defined on the Grothendieck group, $K_0(Q)$, of the algebra kQ, with $k(M)$ being positive on the class of every module. A representation is called μ-stable, respectively μ-semistable, if for all proper non-zero subrepresentations the value which the function μ assumes on it is smaller, respectively smaller or equal. The paper studies this kind of functions on quivers of wild algebras.

The paper considers the following 5 properties: S(1) Each indecomposable preprojective and each indecomposable preinjective is μ-stable. S(2) Let P be a preprojective representation, let R be a regular representation, and let I be a preinjective representation. Then $\mu(P) < \mu(R) < \mu(I)$. S(3) An indecomposable regular representation M is μ-stable precisely if its Aulander-Reiten translation τM is also μ-stable. S(4) Let M be an indecomposable non-projective representation. Then $\mu(\tau M) > \mu(M)$ precisely when M is regular. S(5) Let M and N be representations, both without projective direct summand. Then $\mu(N) < \mu(M)$ if and only if $\mu(\tau N) < \mu(\tau M)$.

These properties are not all independent but they are singled out in the paper. The existence of such a slope is a natural question, but there is a natural candidate for a slope using the Coxeter transformation, this when it exists is called by the authors a distinguished slope.

Properties of a distinguished slope are studied and a necessary and sufficient condition for its existence is given, which uses the maximal and minimal eigenvalue of the Coxeter transformation. One property of the distingued slope is that it satisfies the conditions S(2), S(3), S(4), above. Also two distinguished slopes are equivalent, that is, the set of stable representations is the same for both of them. A necessary and sufficient condition for it to satisfy condition S(1) is given.

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MSC:

16G20 Representations of quivers and partially ordered sets
16G70 Auslander-Reiten sequences (almost split sequences) and Auslander-Reiten quivers
16G60 Representation type (finite, tame, wild, etc.) of associative algebras

Keywords:

quiver representations; distinguished slopes; wild algebras; indecomposable representations; Auslander-Reiten translations; Coxeter transformations