He, Cheng; **Xin, Zhouping**

On the decay properties of solutions to the non-stationary Navier-Stokes equations in $\mathbb{R}^3$.

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The authors investigate the solutions of the non-stationary Navier-Stokes system

$$\frac{\partial u}{\partial t} - \nu\Delta u + (u\cdot\nabla)u = -\nabla p, \quad \text{div } u = 0 \quad \text{in } \mathbb{R}^3 \times (0,\infty),$$

$$u \to 0 \quad \text{as } |x| \to +\infty, \quad u(x,0) = a(x) \quad \text{in } \mathbb{R}^3.$$

Here $u = (u_1, u_2, u_3)$ and p denote the unknown velocity vector and the pressure of the fluid at point $(x,t) \in \mathbb{R}^3 \times (0,\infty)$, respectively, while $\nu > 0$ is the viscosity and $a(x)$ is given initial velocity vector field. For simplicity $\nu = 1$. The asymptotic decay properties in both spatial and temporal variables for a class of weak and strong solutions are studied. The main result is that for the strong solution, the rate of temporal decay depends on the rate of spatial decay of the initial data. Such rates of decay are optimal.

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MSC:

35Q30 Navier-Stokes equations
35B40 Asymptotic behavior of solutions to PDEs
35D05 Existence of generalized solutions of PDE (MSC2000)

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Navier-Stokes system; weak solution; strong solution; weighted spaces; decay properties; temporal decay; spatial decay

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