

**Zhang, Shouwu**

**Heights of Heegner points on Shimura curves.** (English) Zbl 1036.11029  
*Ann. Math. (2)* 153, No. 1, 27-147 (2001).

Let  $F$  be a totally real number field, and let  $N$  be a nonzero ideal of the ring  $\mathcal{O}_F$  of integers in  $F$ . Let  $f$  be a new form on  $\mathrm{GL}_2(\mathbb{A}_F)$  of weight two and level  $K_0(N)$  with trivial central character, where  $K_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{GL}(\mathcal{O}_F \times \prod_p \mathbb{Z}_p) \mid c \in N \times \prod_p \mathbb{Z}_p \right\}$ . Let  $\mathcal{O}_f$  be the subalgebra of  $\mathbb{C}$  over  $\mathbb{Z}$  generated by the eigenvalues  $a(m, f)$  of  $f$  under the Hecke operators  $T(m)$  with  $(m, N) = 1$ . Each embedding  $\sigma : \mathcal{O}_f \rightarrow \mathbb{C}$  determines a new form  $f^\sigma$  such that  $a(f^\sigma, m) = a(f, m)^\sigma$ . Assume that either  $[F : \mathbb{Q}]$  is odd or  $\mathrm{ord}_v(N) = 1$  for at least one finite place  $v$  of  $F$ . Then there exists an abelian variety  $A$  over  $F$  of dimension  $[\mathcal{O}_f : \mathbb{Z}]$  such that its  $L$ -function  $L(s, A)$  coincides with  $\prod_{\sigma: \mathcal{O}_f \rightarrow \mathbb{C}} L(s, f^\sigma)$  modulo the factors at the places dividing  $N$ . In this paper, under the assumption that the  $L$ -function  $L(s, f)$  has order at most one at  $s = 1$ , the author proves that the rank of the Mordell-Weil group  $A(F)$  is equal to  $[\mathcal{O}_f : \mathbb{Z}] \mathrm{ord}_{s=1} L(s, f)$  and that the Shafarevich-Tate group of  $A$  is finite. The proof is carried out by studying Heegner points over an imaginary quadratic extension of  $F$ .

Reviewer: [Min Ho Lee \(Cedar Falls\)](#)

**MSC:**

- [11G18](#) Arithmetic aspects of modular and Shimura varieties
- [14G35](#) Modular and Shimura varieties
- [11F11](#) Holomorphic modular forms of integral weight
- [11G40](#)  $L$ -functions of varieties over global fields; Birch-Swinnerton-Dyer conjecture
- [11G50](#) Heights

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Cited in **55** Documents

**Keywords:**

[Shimura curves](#); [Heegner points](#); [L-functions](#); [modular forms](#); [Hecke operators](#); [Mordell group](#); [Shafarevich-Tate group](#)

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