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Representations of quantum toroidal algebra $U_q(\mathbf{sl}_{n+1,\mathbf{tor}})(n \geq 2)$. (English) Zbl 1028.17011

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From the introduction: After the work of Drinfeld on finite-dimensional representations of the Yangian, V. Chari and A. Pressley studied finite-dimensional representations of the quantum affine algebra in a series of papers. Among their results, those related to our work are as follows. First [*V. Chari* and *A. Pressley*, Commun. Math. Phys. 142, 261-283 (1991; Zbl 0739.17004), A guide to quantum groups (Cambridge University Press) (1994; Zbl 0839.17009) and CMS Conf. Proc. 16, 59-78 (1995; Zbl 0855.17009)] they proved that irreducible finite-dimensional representations are characterizable by Drinfeld polynomials as in the Yangian case. Moreover they showed that the existence of R matrices acting on their tensor products was proven by utilizing the Drinfeld polynomials associated to the tensor products. Then possible $U_q'(\widehat{\mathrm{sl}_{n+1}})$ module structures on irreducible finite-dimensional $U_q(\mathrm{sl}_{n+1})$ modules were shown to be only those via the homomorphisms $U_q'(\widehat{\mathrm{sl}_{n+1}}) \to U_q(\mathrm{gl}_{n+1})$ by Jimbo. Moreover, minimal affinizations of representations of quantum groups of nonaffine type were studied.

In this paper, we apply their method to highest weight representations of the quantum toroidal algebra $U_{q,\kappa}(\mathrm{sl}_{n+1,\mathrm{tor}})$ (κ is the parameter contained in the algebra). Many of the results obtained by Chari and Pressley for $U_q'(\widehat{\mathrm{sl}_{n+1}})$ $(n \geq 2)$ can be generalized to our case almost verbatim. In this analysis, we use the automorphism of the quantum toroidal algebra obtained in [*K. Miki*, Lett. Math. Phys. 47, 365-378 (1999; Zbl 1022.17009)]. Our main results are the proofs of the following facts: (i) some class of irreducible highest weight representations of the quantum toroidal algebra are characterized by Drinfeld polynomials, (ii) there exist solutions of the Yang-Baxter equation which depend on a spectral parameter and act on the tensor product of irreducible highest weight representations characterized by Drinfeld polynomials, (iii) no toroidal action can be defined on integrable highest weight representations of $U_q(\widehat{\mathrm{sl}_{n+1}})$ with level > 1, (iv) if the parameter κ is not equal to $q^{\pm(n+1)}$, then toroidal module structures can be defined on irreducible integrable highest weight representations of $U_q(\widehat{\mathrm{gl}_{n+1}})$ with level $c > 1$ if and only if $\kappa = q^{\pm(n+1+2c)}$. Moreover, these structures are those via the homomorphisms from the quantum toroidal algebra to a completion of $U_q(\widehat{\mathrm{gl}_{n+1}})$.

Note that our result clarifies the relation between the level 1 representation of the quantum toroidal algebra of Y. Saito (1998) and the one by M. Varagnolo and E. Vasserot (1998) and Y. Saito, K. Takemura and D. Uglov (1998).

MSC:

17B37 Quantum groups (quantized enveloping algebras) and related deformations
81R50 Quantum groups and related algebraic methods applied to problems in quantum theory

Keywords:

Yangian; quantum affine algebra; minimal affinizations; quantum groups; highest weight representations; quantum toroidal algebra; Drinfeld polynomials; Yang-Baxter equation; integrable highest weight representations; level 1 representation

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