Atiyah, Michael
Geometry and physics in the 20th century. (English) Zbl 0994.01011
Bitar, Khalil (ed.) et al., Proceedings of the international conference on the mathematical sciences after the year 2000, Beirut, Lebanon, January 11-15, 1999. Singapore: World Scientific. 1-9 (2000).

This is a brief survey of the striking and totally unexpected new interaction between geometry and quantum theory during the last 25 years. After a description of the classical era and the quantum era the author gives short summaries of the essential features of quantum cohomology, Jones-Witten invariants, Donaldson invariants, and topological quantum field theories. Then he deals with the question what we should expect in the 21st century. The reader is warned that the biggest break-throughs cannot be predicted and there will inevitably be surprises. Nevertheless one can identify the following three different philosophical strands: (1) the main physics community led by Witten which sees string theory (or its new off-shoot M-theory) as the way forward; (2) the noncommutative geometry of Alain Connes; (3) Penrose's view that some new physical insight is required which will alter our fundamental approach.

It is possible that these three scenarios will merge in some way to give different aspects of the same reality.

For the entire collection see [Zbl 0966.00022].

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MSC:

01A67	Future perspectives in mathematics
81-03	History of quantum theory
53-03	History of differential geometry
58-03	History of global analysis
01A60	History of mathematics in the 20th century
01A61	History of mathematics in the 21st century
81T75	Noncommutative geometry methods in quantum field theory
58B34	Noncommutative geometry (à la Connes)
53D45	Gromov-Witten invariants, quantum cohomology, Frobenius manifolds
81T45	Topological field theories in quantum mechanics
57R57	Applications of global analysis to structures on manifolds
81T30	String and superstring theories; other extended objects (e.g., branes) in quantum field theory

Keywords:

brief survey; interaction between geometry and quantum theory; quantum cohomology; Jones-Witten invariants; Donaldson invariants; topological quantum field theories