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**Magnetic bottles in connection with superconductivity.** (English) Zbl 1078.81023  
*J. Funct. Anal.* 185, No. 2, 604-680 (2001).

Let  $\Omega \subset \mathbb{R}^2$  be an open set, and let  $P_{h,A,\Omega} = (hD_{x_1} - A_1)^2 + (hD_{x_2} - A_2)^2$ , where  $h > 0$  is a small parameter. In this paper, the authors treat in a systematic way and sharpen some results present in the literature on the lowest eigenvalue of  $P_{h,A,\Omega}$  in the Dirichlet and Neumann realizations, respectively, and give accurate estimates on the localization of the ground-state in the case of the Neumann realization. In particular, they prove the following result conjectured by *A. Bernoff* and *P. Sternberg* [*J. Math. Phys.* 39, 1272–1284 (1998; [Zbl 1056.82523](#))]. Theorem: Suppose that the magnetic field  $B = \partial_{x_1} A_2 - \partial_{x_2} A_1$  is a non-zero constant. Then any normalized ground-state of the Neumann realization of  $P_{h,A,\Omega}$  is exponentially localized as  $h \rightarrow 0$  in the neighborhood of the points of the boundary  $\partial\Omega$  with maximal curvature.

In the final section of the paper, they indicate a number of interesting open problems.

Reviewer: [Alberto Parmeggiani \(Bologna\)](#)

**MSC:**

- [81Q10](#) Selfadjoint operator theory in quantum theory, including spectral analysis
- [81Q20](#) Semiclassical techniques, including WKB and Maslov methods applied to problems in quantum theory
- [35J10](#) Schrödinger operator, Schrödinger equation
- [35P15](#) Estimates of eigenvalues in context of PDEs
- [35Q40](#) PDEs in connection with quantum mechanics
- [82D55](#) Statistical mechanical studies of superconductors

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**Keywords:**

[Schrödinger operators](#); [semiclassical analysis](#); [lowest eigenvalue](#); [localization of eigenfunctions](#)

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**References:**

- [1] Agmon, S., Lectures on exponential decay of solutions of second order elliptic equations, *Math. notes*, 29, (1982), Princeton University Press Princeton · [Zbl 0503.35001](#)
- [2] Avron, J.; Herbst, I.; Simon, B., Schrödinger operators with magnetic fields I, *Duke math. J.*, 45, 847-883, (1978) · [Zbl 0399.35029](#)
- [3] Bauman, P.; Phillips, D.; Tang, Q., Stable nucleation for the ginzburg – landau system with an applied magnetic field, *Arch. rational mech. anal.*, 142, 1-43, (1998) · [Zbl 0922.35157](#)
- [4] Y. Belaud, B. Helffer, and, L. Véron, Long-time vanishing properties of solutions of some semi-linear parabolic equations, *Ann. Inst. H. Poincaré Anal. Non Linéaire*, in press.
- [5] Bernoff, A.; Sternberg, P., Onset of superconductivity in decreasing fields for general domains, *J. math. phys.*, 39, 1272-1284, (1998) · [Zbl 1056.82523](#)
- [6] C. Bolley, Modélisation du champ de retard à la condensation d'un supraconducteur par un problème de bifurcation, *\textit{M}2AN*26, 1992, 235-287. · [Zbl 0741.35085](#)
- [7] Bolley, C.; Helffer, B., An application of semi-classical analysis to the asymptotic study of the supercooling field of a superconducting material, *Ann. inst. H. Poincaré phys. théor.*, 58, 169-233, (1993) · [Zbl 0779.35104](#)
- [8] Chapman, S.J., Nucleation of superconductivity in decreasing fields, *European J. appl. math.*, 5, 449-468, (1994) · [Zbl 0820.35124](#)
- [9] Cycon, H.L.; Froese, R.G.; Kirsch, W.; Simon, B., *Schrödinger operators*, (1987), Springer-Verlag Berlin
- [10] Dauge, M.; Helffer, B., Eigenvalues variation I, Neumann problem for sturm – liouville operators, *J. differential equations*, 104, 243-262, (1993) · [Zbl 0784.34021](#)
- [11] De Bièvre, S.; Pulé, J.V., Propagating edge states for a magnetic Hamiltonian, *Math. phys. electr. J.*, 5, (1999) · [Zbl 0930.35144](#)

- [12] Erdős, L., Rayleigh-type isoperimetric inequality with a homogeneous magnetic field, *Calc. var. partial differential equations*, 4, 283-292, (1996) · [Zbl 0846.35094](#)
- [13] Erdős, L., Lifschitz tail in a magnetic field: the non classical regime, *Probab. theor. related fields*, 112, 321-371, (1998) · [Zbl 0921.60099](#)
- [14] Fröhlich, J.; Graf, G.M.; Walcher, J., Extended quantum Hall edge states. general domains, *Ann. H. Poincaré*, 1, 405-442, (2000) · [Zbl 1004.81043](#)
- [15] Giorgi, T.; Phillips, D., The breakdown of superconductivity due to strong fields for the ginzburg – landau model, *SIAM J. math. anal.*, 30, 341-359, (1999) · [Zbl 0920.35058](#)
- [16] Helffer, B., *Introduction to the semiclassical analysis for the Schrödinger operator and applications*, Springer lecture notes in math., 1336, (1988), Springer-Verlag Berlin/New York
- [17] Helffer, B., On spectral theory for Schrödinger operators with magnetic potentials, *Adv. studies pure math.*, 23, 113-141, (1993) · [Zbl 0816.35100](#)
- [18] Helffer, B., Semi-classical analysis for the Schrödinger operator with magnetic wells (after R. Montgomery, B. helffer-A. mohamed), *Proceedings of the conference in Minneapolis, the IMA volumes in mathematics and its applications*, vol. 95, quasiclassical methods, (1997), Springer-Verlag Berlin/New York, p. 99-114 · [Zbl 0887.35131](#)
- [19] Helffer, B.; Hoffmann-Ostenhof, T.; Hoffmann-Ostenhof, M.; Owen, M., Nodal sets for the groundstate of the Schrödinger operator with zero magnetic field in a non simply connected domain, *Comm. math. phys.*, 202, 629-649, (1999) · [Zbl 1042.81012](#)
- [20] Helffer, B.; Mohamed, A., Sur le spectre essentiel des opérateurs de Schrödinger avec champ magnétique, *Ann. inst. Fourier*, 38, 95-113, (1988) · [Zbl 0638.47047](#)
- [21] Helffer, B.; Mohamed, A., Semiclassical analysis for the ground state energy of a Schrödinger operator with magnetic wells, *J. funct. anal.*, 138, 40-81, (1996) · [Zbl 0851.58046](#)
- [22] Helffer, B.; Nourrigat, J., Hypocoellipticité maximale pour des opérateurs polynômes de champs de vecteurs, (1985), Birkhäuser Boston · [Zbl 0549.35026](#)
- [23] Helffer, B.; Nourrigat, J., Décroissance à l'infini des fonctions propres de l'opérateur de Schrödinger avec champ électromagnétique polynomial, *J. anal. math. Jérusalem*, 58, 263-275, (1992) · [Zbl 0814.35080](#)
- [24] Helffer, B.; Robert, D., Puits de potentiel généralisés et asymptotique semi-classique, *Ann. inst. H. Poincaré phys. théor.*, 41, 291-331, (1984) · [Zbl 0565.35082](#)
- [25] Helffer, B.; Sjöstrand, J., Multiple wells in the semiclassical limit I, *Comm. partial differential equations*, 9, 337-408, (1984) · [Zbl 0546.35053](#)
- [26] Helffer, B.; Sjöstrand, J., Puits multiples en limite semiclassique V—le cas des minipuits—, (), 133-186
- [27] Helffer, B.; Sjöstrand, J., Puits multiples en limite semi-classique VI—le cas des puits variétés—, *Ann. inst. H. Poincaré phys. théor.*, 46, 353-373, (1987) · [Zbl 0648.35027](#)
- [28] Helffer, B.; Sjöstrand, J., Effet tunnel pour l'équation de Schrödinger avec champ magnétique, *Ann. ENS pise*, XIV, 625-657, (1987) · [Zbl 0699.35205](#)
- [29] Hornberger, K.; Smilansky, U., The boundary integral method for magnetic billiards, *J. phys. A*, 33, 2829-2855, (2000) · [Zbl 0954.81018](#)
- [30] K. Hornberger, and, U. Smilansky, The exterior and interior edge states of magnetic billiards, preprint, September 2000. · [Zbl 0954.81018](#)
- [31] H. T. Jadallah, The onset of superconductivity in a domain with a corner, preprint, August 2000. · [Zbl 1063.82041](#)
- [32] Lu, K.; Pan, X.-B., Estimates of the upper critical field for the ginzburg – landau equations of superconductivity, *Physica D*, 127, 73-104, (1999) · [Zbl 0934.35174](#)
- [33] Lu, K.; Pan, X.-B., Eigenvalue problems of ginzburg – landau operator in bounded domains, *J. math. phys.*, 40, 647-2670, (1999) · [Zbl 0943.35058](#)
- [34] Lu, K.; Pan, X.-B., Gauge invariant eigenvalue problems on  $\mathbb{R}^2$  and  $\mathbb{R}^2_+$ , *Trans. amer. math. soc.*, 352, 1247-1276, (2000) · [Zbl 1053.35124](#)
- [35] K. Lu, and, X.-B. Pan, Ginzburg-Landau system and surface nucleation of superconductivity, *Proceeding of the IMS Workshop on Reaction-Diffusion systems*, Chinese University of Hong-Kong, December 6-11, 1999.
- [36] Lu, K.; Pan, X.-B., Surface nucleation of superconductivity in 3-dimension, *J. differential equations*, 168, 386-452, (2000) · [Zbl 0972.35152](#)
- [37] L. Maigrot, Encadrement de la première valeur propre d'un opérateur de Schrödinger dégénéré, preprint, University of Reims, June 1999.
- [38] A. Martinez and V. Sordani, Microlocal WKB expansions, *Math. Phys.*, preprint Archive, 98-203, [\textit{www.ma.utexas.edu/mp\\_arc/}](http://www.ma.utexas.edu/mp_arc/). · [Zbl 0941.35136](#)
- [39] Matsumoto, H., Semiclassical asymptotics of eigenvalues for Schrödinger operators with magnetic fields, *J. functional anal.*, 129, 168-190, (1995) · [Zbl 0859.35081](#)
- [40] Matsumoto, H.; Ueki, N., Spectral analysis of Schrödinger operators with magnetic fields, *J. functional anal.*, 140, 218-255, (1996) · [Zbl 0866.35083](#)
- [41] Melin, A., Lower bounds for pseudo-differential operators, *Ark. math.*, 9, 117-140, (1971) · [Zbl 0211.17102](#)
- [42] Mohamed, A.; Nourrigat, J., Encadrement du  $\text{N}(\lambda)$  pour un opérateur de Schrödinger avec un champ magnétique et un potentiel électrique, *J. math. pures appl.*, 70, 87-99, (1991) · [Zbl 0725.35068](#)

- [43] Mohamed, A.; Parisse, B., Approximation des valeurs propres de certaines perturbations singulières et application à l'opérateur de Dirac, *Ann. inst. H. Poincaré phys. théor.*, 56, 235-277, (1992) · [Zbl 0755.35106](#)
- [44] Montgomery, R., Hearing the zero locus of a magnetic field, *Comm. math. phys.*, 168, 651-675, (1995) · [Zbl 0827.58076](#)
- [45] X.-B. Pan, Upper critical for superconductors with edges and corners, preprint, 2000, submitted for publication.
- [46] B. Parisse, Construction BKW en fonds de puits, cas particuliers, (Schrödinger, Dirac avec champ magnétique), preprint, Université de Grenoble, December 1999.
- [47] del Pino, M.; Felmer, P.L.; Sternberg, P., Boundary concentration for eigenvalue problems related to the onset of superconductivity, *Comm. math. phys.*, 210, 413-446, (2000) · [Zbl 0982.35077](#)
- [48] Reed, M.; Simon, B., *Methods of modern mathematical physics, IV: analysis of operators*, (1978), Academic Press New York · [Zbl 0401.47001](#)
- [49] Shigekawa, I., Eigenvalue problems for the Schrödinger operator with the magnetic field on a compact Riemannian manifold, *J. funct. anal.*, 75, (1987) · [Zbl 0629.58023](#)
- [50] Simon, B., Semi-classical analysis of low lying eigenvalues I, *Ann. inst. H. Poincaré*, 38, 295-307, (1983)
- [51] Sordani, V., Gaussian decay for the eigenfunctions of a Schrödinger operator with magnetic field constant at infinity, *Comm. partial differential equations*, 23, 223-242, (1998) · [Zbl 0898.34077](#)
- [52] Ueki, N., Lower bounds for the spectra of Schrödinger operators with magnetic fields, *J. funct. anal.*, 120, 344-379, (1994) · [Zbl 0805.35025](#)
- [53] Ueki, N., Asymptotics of the infimum of the spectrum of Schrödinger operators with magnetic fields, *J. math. Kyoto univ.*, 37, 615-638, (1998) · [Zbl 0928.35032](#)

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