Harris, Michael; Taylor, Richard

**The geometry and cohomology of some simple Shimura varieties. With an appendix by Vladimir G. Berkovich.** (English) Zbl 1036.11027

Annals of Mathematics Studies 151. Princeton, NJ: Princeton University Press (ISBN 0-691-09090-4/hbk; 0-691-09092-0/pbk). viii, 276 p. (2001).

The goal of this book is to prove the local Langlands conjecture for $\mathrm{GL}_n$ over a $p$-adic field and identify the action of the decomposition group at a prime of bad reduction on the $\ell$-adic cohomology of some simple Shimura varieties. Let $K$ be a finite extension of $\mathbb{Q}_p$, and let $W_K$ denotes its Weil group. Then there is canonical isomorphism $\mathrm{Art}_K : K^\times \to W_K^{\mathrm{ab}}$, and the local Langlands conjecture provides a description of $W_K$. Let $\mathrm{Irr}(\mathrm{GL}_n(K))$ denote the set of isomorphism classes of irreducible admissible representations of $\mathrm{GL}_n(K)$ over $\mathbb{C}$, and let $\mathrm{WDRep}_n(W_K)$ denote the set of isomorphism classes of $n$-dimensional Frobenius semisimple Weil-Deligne representations of $W_K$ over $\mathbb{C}$. A local Langlands correspondence for $K$ is a collection of bijections

$$\mathrm{rec}_K : \mathrm{Irr}(\mathrm{GL}_n(K)) \to \mathrm{WDRep}_n(W_K)$$

for all $n \geq 1$ satisfying a certain set of conditions, and the local Langlands conjecture states that $\mathrm{rec}_K$ exists for any finite extension $K$ of $\mathbb{Q}_p$. The authors prove this conjecture by constructing the maps

$$\mathrm{rec}_K : \mathrm{Cusp}(\mathrm{GL}_n(K)) \to \mathrm{Irr}_n(W_K)$$

satisfying certain properties, where $\mathrm{Cusp}(\mathrm{GL}_n(K))$ is the subset of $\mathrm{Irr}(\mathrm{GL}_n(K))$ consisting of equivalence classes of supercuspidal representations and $\mathrm{Irr}_n(W_K)$ denotes the subset of $\mathrm{WDRep}_n(W_K)$ consisting of equivalence classes of pairs $(r, 0)$ with $r$ irreducible. Their methods show that the local reciprocity map $\mathrm{rec}_K$ is compatible with global reciprocity maps in some cases, and one of the key ingredients of their construction is an analysis of the bad reduction of certain simple Shimura varieties.

Another proof of the local Langlands conjecture was given by *G. Henniart* [Invent. Math. 139, 439–455 (2000; Zbl 1048.11092)].

Reviewer: Min Ho Lee (Cedar Falls)

**MSC:**

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|---|---|
| 11G18 | Arithmetic aspects of modular and Shimura varieties |
| 14G35 | Modular and Shimura varieties |
| 11F70 | Representation-theoretic methods; automorphic representations over local and global fields |
| 11-02 | Research exposition (monographs, survey articles) pertaining to number theory |
| 14-02 | Research exposition (monographs, survey articles) pertaining to algebraic geometry |
| 22E45 | Representations of Lie and linear algebraic groups over real fields: analytic methods |
| 11S37 | Langlands-Weil conjectures, nonabelian class field theory |



**Keywords:**

Shimura varieties; local Langlands conjecture; automorphic representations; supercuspidal representations

**Full Text:** DOI