

Sgibnev, M. S.

Stone's decomposition of the renewal measure via Banach-algebraic techniques. (English)

Zbl 0996.60096

Proc. Am. Math. Soc. 130, No. 8, 2425-2430 (2002).

Let F be a probability distribution on \mathbb{R} with positive mean μ and let H be the corresponding renewal measure. *C. Stone* [Ann. Math. Stat. 37, 271-275 (1966; Zbl 0147.16205)] showed that, if for some $m \geq 1$ m -times convolution of F has a nonzero absolutely continuous component, then there exists a decomposition $H = H_1 + H_2$, where H_2 is a finite measure and H_1 is absolutely continuous with bounded continuous density $h(x)$ such that $\lim_{x \rightarrow +\infty} h(x) = \mu^{-1}$ and $\lim_{x \rightarrow -\infty} h(x) = 0$. A lot of estimations are based on the representation of H under some additional assumptions. Stone's decomposition is proved by using Banach-algebraic techniques. The method allows to extract detailed information about the asymptotic properties of the terms H_1 and H_2 . Under some additional restrictions of submultiplicative type, estimates of the rate of convergence in the key renewal theorem are obtained.

Reviewer: [Valentin Topchii \(Omsk\)](#)

MSC:

[60K05](#) Renewal theory

Cited in **2** Reviews
Cited in **5** Documents

Keywords:

Stone's decomposition; renewal measure; asymptotic behavior; submultiplicative function; spread-out distribution; Banach algebra

Full Text: [DOI](#)

References:

- [1] Gerold Alsmeyer, Erneuerungstheorie, Teubner Skripten zur Mathematischen Stochastik. [Teubner Texts on Mathematical Stochastics], B. G. Teubner, Stuttgart, 1991 (German). Analyse stochastischer Regenerationsschemata. [Analysis of stochastic regeneration schemes]. · [Zbl 0727.60102](#)
- [2] Elja Arjas, Esa Nummelin, and Richard L. Tweedie, Uniform limit theorems for non-singular renewal and Markov renewal processes, J. Appl. Probability 15 (1978), no. 1, 112 – 125. · [Zbl 0375.60095](#)
- [3] William Feller, An introduction to probability theory and its applications. Vol. II, John Wiley & Sons, Inc., New York-London-Sydney, 1966. · [Zbl 0077.12201](#)
- [4] Rudolf Grübel, On subordinated distributions and generalized renewal measures, Ann. Probab. 15 (1987), no. 1, 394 – 415. · [Zbl 0613.60007](#)
- [5] E. Hille, R. S. Phillips, Functional Analysis and Semi-Groups, Amer. Math. Soc. Colloquium Publications, vol. 31, Providence, RI, 1957. · [Zbl 0078.10004](#)
- [6] B. A. Rogozin, Asymptotic analysis of the renewal function, Teor. Veroyatnost. i Primenen. 21 (1976), no. 4, 689 – 706 (Russian, with English summary).
- [7] B. A. Rogozin and M. S. Sgibnev, Banach algebras of measures on the line, Sibirsk. Mat. Zh. 21 (1980), no. 2, 160 – 169, 239 (Russian). · [Zbl 0457.46024](#)
- [8] Manfred Schäl, Über Lösungen einer Erneuerungsgleichung, Abh. Math. Sem. Univ. Hamburg 36 (1971), 89 – 98 (German). Collection of articles dedicated to Lothar Collatz on his sixtieth birthday. · [Zbl 0218.60086](#) · [doi:10.1007/BF02995911](#) · [doi.org](#)
- [9] M. S. Sgibnev, Submultiplicative moments of the supremum of a random walk with negative drift, Statist. Probab. Lett. 32 (1997), no. 4, 377 – 383. · [Zbl 0903.60055](#) · [doi:10.1016/S0167-7152\(96\)00097-1](#) · [doi.org](#)
- [10] Mikhail S. Sgibnev, Exact asymptotic behaviour in a renewal theorem for convolution equivalent distributions with exponential tails, SUT J. Math. 35 (1999), no. 2, 247 – 262. · [Zbl 0958.60083](#)
- [11] M. S. Sgibnev, An asymptotic expansion for the distribution of the supremum of a random walk, Studia Math. 140 (2000), no. 1, 41 – 55. · [Zbl 0962.60019](#)
- [12] W. L. Smith, Regenerative stochastic processes. Proc. R. Soc. London A 232 (1955), 6-31. · [Zbl 0067.36301](#)
- [13] W. L. Smith, Remarks on the paper 'Regenerative stochastic processes', Proc. Roy. Soc. London. Ser. A 256 (1960), 496 –

501. · Zbl 0119.14203 · doi:10.1098/rspa.1960.0121 · doi.org

- [14] Charles Stone, On absolutely continuous components and renewal theory, *Ann. Math. Statist.* 37 (1966), 271 – 275. · Zbl 0147.16205 · doi:10.1214/aoms/1177699617 · doi.org
- [15] N. B. Engibaryan, Renewal equations on the half-line, *Izv. Ross. Akad. Nauk Ser. Mat.* 63 (1999), no. 1, 61 – 76 (Russian, with Russian summary); English transl., *Izv. Math.* 63 (1999), no. 1, 57 – 71. , <https://doi.org/10.1070/im1999v063n01ABEH000228>
Norair B. Yengibarian, Renewal equation on the whole line, *Stochastic Process. Appl.* 85 (2000), no. 2, 237 – 247. · Zbl 0997.60096 · doi:10.1016/S0304-4149(99)00076-9 · doi.org

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.