Faris, William G.
Ornstein-Uhlenbeck and renormalization semigroups. (English) ❘Zbl 1022.81042❘
Mosc. Math. J. 1, No. 3, 389-405 (2001).

An abstract characterization of the Ornstein-Uhlenbeck semigroup corresponding to the renormalization group realized as a combination of a scaling with a Gaussian convolution is given. Precisely, let $H = L^2(\mathbb{R}^n, d^n x)$ be the Hilbert space, $n > 2$ and $D = x \cdot \nabla + \nu/2$, $-A = D - 1$. Let $\exp(-tL)$ be the Uhlenbeck-Ornstein semigroup corresponding to the renormalization group, with flow $\exp(-tA)$ generated by $-A$. Let ν be the invariant Gaussian measure with covariance $G = C + aS$, $C = (-\Delta)^{-1}(1 - \exp(\alpha\Delta))$, $S = (-\Delta)^{-1}$, $a > 0$, where α determines the cutoff. Then it is proved $G = RS$, where R, S commute and R is selfadjoint and invertible. Furthermore, the Hilbert space adjoint semigroup $\exp(-tL^*)$ is the Ornstein-Uhlenbeck semigroup with flow $G \exp(-tA^*)G^{-1} = R \exp(tA)R^{-1}$ generated by $-GA^*G^{-1} = RAR^{-1}$ (Theorem 7.1).

To prove the theorem, first measures on Hilbert space is reviewed following *N. N. Vakhaniya, V. I. Tarieladze* and *S. A. Chobanyan* [Probability distributions on Banach spaces, Dordrecht (1987; Zbl 0698.60003)], in Sect. 2. Ornstein-Uhlenbeck semigroup (Mehler semigroup) is reviewed in Sect. 3 following *V. I. Bogachev, M. Röckner* and *B. Schmuland*, [Generalized Mehler semigroups and applications, Probab. Theory Related Fields 105, 193-225 (1996; Zbl 0849.60066)]. Sect. 4 and 5 are devoted to the study of the generator of the Ornstein-Uhlenbeck semigroup and the adjoint semigroup. Then abstract renormalization group is treated in Sect. 6. After these preparations, Theorem 7.1 is proved. The RG example suggests the program of looking for other pairs consisting of a diffusion given by Q and a linear vector field. One such example taking the covariance Q to be an integral operator was given by *G. Da Prato* and *J. Zabczyk* [Stochastic equations in infinite dimensions, Encyclopedia of Mathematics and its Applications, 44. Cambridge (1992; Zbl 0761.60052)]. This example and its generalization are discussed in Sect. 8 and 9.

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MSC:

81T17	Renormalization group methods applied to problems in quantum field theory	Cited in **1** Document
82B28	Renormalization group methods in equilibrium statistical mechanics	
47D06	One-parameter semigroups and linear evolution equations	
60J60	Diffusion processes	

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