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Introduction to quantum groups and crystal bases. (English) Zbl 1134.17007

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This fine introduction to quantum groups and crystal bases is best described by the authors' well written summary:

“The notion of a “quantum group” was introduced by V. G. Dinfeld and M. Jimbo, independently, in their study of the quantum Yang-Baxter equation arising from 2-dimensional solvable lattice models. Quantum groups are certain families of Hopf algebras that are deformations of universal enveloping algebras of Kac-Moody algebras. And over the past 20 years, they have turned out to be the fundamental algebraic structure behind many branches of mathematics and mathematical physics, such as solvable lattice models in statistical mechanics, topological invariant theory of links and knots, representation theory of Kac-Moody algebras, representation theory of algebraic structures, topological quantum field theory, geometric representation theory, and C^* -algebras.

In particular, the theory of “crystal bases” or “canonical bases” developed independently by M. Kashiwara and G. Lusztig provides a powerful combinatorial and geometric tool to study the representations of quantum groups. The purpose of this book is to provide an elementary introduction to the theory of quantum groups and crystal bases, focusing on the combinatorial aspects of the theory.”

The authors start with the basic theory of quantum groups and their representations, and then give a detailed exposition of the fundamental features of crystal basis theory. They also discuss its applications to the representation theory of classical Lie algebras and quantum affine algebras, solvable lattice model theory, and combinatorics of Young walls.

Contents: 1. Lie algebras and Hopf algebras, 2. Kac-Moody algebras, 3. Quantum groups, 4. Crystal bases, 5. Existence and uniqueness of crystal bases, 6. Global bases, 7. Young tableaux and crystals, 8. Crystal graphs for classical Lie algebras, 9. Solvable lattice models, 10. Perfect crystals, 11. Combinatorics of Young walls.

Reviewer: [Olaf Ninnemann \(Berlin\)](#)

MSC:

- 17B37** Quantum groups (quantized enveloping algebras) and related deformations
- 17-02** Research exposition (monographs, survey articles) pertaining to nonassociative rings and algebras
- 05E15** Combinatorial aspects of groups and algebras (MSC2010)
- 81R50** Quantum groups and related algebraic methods applied to problems in quantum theory
- 82B23** Exactly solvable models; Bethe ansatz

Cited in **3** Reviews
Cited in **186** Documents