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A Morse lemma for degenerate critical points with low differentiability. (English)

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Abstr. Appl. Anal. 5, No. 2, 113-118 (2000).

Let $f : U \subset \mathbb{H} \to \mathbb{R}$ be a C^1 function defined on an open set U of a Hilbert space $\mathbb{H}$. If f is twice differentiable at 0 and $A : \mathbb{H} \to \mathbb{H}$ the symmetric operator defined by $\langle Av, u\rangle = \frac{1}{2} d^2 f_0(u, v)$ then $\mathbb{H} = N^\perp \oplus N$, where $N = \mathrm{Ker}(A)$.

One proves the following theorem: If f' is strongly differentiable at the origin, there is a neighborhood V of 0 in $\mathbb{H}$ and a homeomorphism $\varphi : V \to \varphi(V) \subset \mathbb{H}$ such that

$$f\big(\varphi(x, y)\big) = \frac{1}{2}\langle Ax, x\rangle + f\big(g(y), y\big),\ d\varphi_0 = I,$$

where g is a function $g : V \cap \mathbb{N} \to \mathbb{N}^\perp$.

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MSC:

58E05 Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces

Keywords:

twice strongly differentiable function; degenerate critical point; Hilbert space; symmetric operator

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