Tzirtzilakis, E.; **Xenos, M.**; **Marinakis, V.**; **Bountis, T. C.**
Interactions and stability of solitary waves in shallow water. (English) Zbl 1068.76011
Chaos Solitons Fractals 14, No. 1, 87-95 (2002).

Summary: We use spectral methods of numerical analysis to study solitary wave solutions of a nonlinear partial differential equation (PDE), which is non-integrable and has been proposed as an improved approximation of shallow water wave propagation compared with the KdV equation. For sufficiently small parameters its solitary waves appear to be stable under time evolution and interact elastically as if they were pure solitons. This behaviour is probably due to the fact that this non-integrable PDE can be transformed to an integrable equation with the aid of a nonlinear local transformation. As in the case of the KdV equation, when their speed increases, these wave solutions become unstable. However, unlike the KdV, the solitary waves of this new PDE, in general, require a non-zero background which implies that they have infinite energy and thus may be unphysical. For any given values of the equation parameters, these waves tend to zero exponentially at infinity, and thus represent a continuation of KdV solitons, only for one value of their velocity.

MSC:

76B25 Solitary waves for incompressible inviscid fluids
76M22 Spectral methods applied to problems in fluid mechanics
35Q51 Soliton equations

Full Text: DOI

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