Moroz, M. V.

On the existence of periodic solutions to a system of two differential equations with pulse influence. (English. Russian original) Zbl 1015.34034

Ukr. Math. J. 54, No. 1, 160-165 (2002); translation from Ukr. Mat. Zh. 54, No. 1, 133-137 (2002).

The author studies the following 2-dimensional system:

$$\dot{x} = Jx \text{ while } \langle x, g\rangle + c > 0; \quad x(t+0) - x(t-0) = h, \text{ if } \langle x(t-0), g\rangle + c = 0. \tag{1}$$

Here, $\langle \cdot, \cdot \rangle$ denotes the scalar product in the coordinate space $\mathbb{R}^2$, $g, h \in \mathbb{R}^2$ are given vectors satisfying the condition $\langle g, h\rangle > 0$, $c \in \mathbb{R}$ is a given number, and J is a 2×2-dimensional Jordan matrix whose eigenvalues have negative real parts.

It is obvious that if, for a point x_0 belonging to the line $L := \{x \in \mathbb{R}^2 : \langle x, g\rangle + c = 0\}$, there exists a $T > 0$ such that $e^{TJ}(x_0 + h) = x_0$ and $\langle e^{TJ}(x_0 + h), g\rangle + c > 0$, $t \in (0, T)$, then the point x_0 gives raise to a T-periodic solution to system (1). Basing on this fact, in order to find periodic solutions to (1), the author seeks sufficient conditions for the solvability of the system $\{\langle x, g\rangle + c = 0, x = e^{JT}(x + h)\}$ with respect to the unknowns x, T. These conditions are expressed in terms of eigenvalues of J and the angle between the vector h and one of the basis vectors of $\mathbb{R}^2$.

Reviewer: I.O.Parasyuk (Kyïv)

MSC:

34C25 Periodic solutions to ordinary differential equations
34A37 Ordinary differential equations with impulses

Keywords:

system with pulse action; periodic solution; fixed point

Full Text: DOI