

Cohn, P. M.

Basic algebra. Groups, rings and fields. (English) [Zbl 1003.00001](#)

London: Springer. xii, 465 p. i EUR 64.95/net; sFr. 108.00; £45.00; \$ 74.95 (2003).

P. M. Cohn's comprehensive text "Algebra", which consists of three volumes, is widely regarded as one of the most outstanding introductory algebra textbook. These three volumes were first published in the 1970's ([Zbl 0272.00003](#) and [Zbl 0341.00002](#)) and, due to their popularity, have seen their second edition around 1990.

Now, more than ten years later, they are out of print, again, but the text is still as popular and used as ever since its first appearance. Instead of preparing a third edition of these three volumes, the author has decided – following an offer by Springer Verlag – to bring out a new version of this entire text, this time in only two volumes and in reworked and updated fashion.

The book under review is the first volume of this new, reworked introductory algebra text. It is entitled "Basic algebra", whereas the second volume will appear soon under the title "Further algebra and applications" (Springer, 2003)(see [Zbl 1006.00001](#)).

As to this first volume under review, the material covered here is based on the author's classic texts "Algebra, vol. 2" (2nd edition, Wiley, London) (1989; [Zbl 0703.00002](#)) and "Algebra, vol. 3" (2nd edition, Wiley, London) (1991; [Zbl 0719.00002](#)), complemented by the definitions and basic facts on groups and rings. According to the new arrangement of the entire text, this first volume covers those basic facts from modern algebra that are usually taught in a second and third year undergraduate course in mathematics, together with some more advanced topics for graduate students. Thus the work is addressed to students who have some knowledge of linear algebra and who have already met groups and fields, though all the essential facts are recalled here. As the author points out, his overall aim is to present as many of the important results in modern algebra as would conveniently fit into a volume of less than 500 pages. Following this distinct strategy, this introductory text also provides a number of concepts and topics that are possibly regarded as less central but nevertheless have many uses and applications, and which are barely found in other comprehensive introductions to modern algebra. This particular fact, apart from clarity, profundity, rigor, and comprehensiveness characterizing the author's writing style, is one of the main features of this textbook and makes it a very special one in its own way.

As to the contents of the book, there are eleven chapters covering the following topics:

Chapter 1 discusses those facts from set theory that are crucial in algebra: countable and uncountable sets, Zorn's lemma and well-ordered sets, graphs, and the beginnings of Ramsey theory.

Chapter 2 presents the basics of group theory, including a nice discourse on the Frattini subgroup and the Fitting subgroup.

Chapter 3 is devoted to modular and distributive lattices, general chain conditions, Boolean algebras, and the basic definitions in category theory.

Chapter 4 is entitled "Rings and modules" and covers, apart from the basics, such special topics as semisimple modules, matrix rings, duality of finite abelian groups, and some elementary homological algebra (projective and injective modules).

Chapter 5 treats algebras, together with such important extras as the regular representation of a finite-dimensional algebra over a field, norm and trace, and Möbius functions. Of course, the Wedderburn structure theorems for algebras, the links with Artin rings, and tensor products of algebras are among the central part of this chapter.

Chapter 6 introduces graded rings, graded modules and graded algebras, discusses Hilbert series in this context, and covers from multilinear algebra the basics on tensor algebras and exterior algebras of modules.

Chapter 7 comes with the title "Field theory" and does the classic things: field extensions, the algebraic closure of a field, separability, roots of unity, finite fields, primitive elements, and elementary Galois theory.

Chapter 8 offers many important extras. Under the title "Quadratic forms and ordered fields" the author

discusses, apart from the indispensable basic facts, also Clifford algebras and the spinor norm, Witt's cancellation theorem and the Witt group of a field, formally real fields, the Witt ring of a field, symplectic groups, and quadratic forms in characteristic two.

Chapter 9 turns to the arithmetical aspects of ring theory and deals with divisibility, valuations, and integral extensions of rings. This includes the discussion of absolute values, the field of p -adic numbers, general valuation rings, extensions of valuations, and Hensel's lemma.

Chapter 10 gives the first fundamental concepts of commutative algebra. Basic ideal theory, localization, Noetherian rings, primary decomposition for Noetherian modules, Dedekind domains and their module theory, affine algebraic sets, the first steps into the dimension theory of rings, and Hilbert's Nullstellensatz are the main topics treated here.

The final Chapter 11 returns to field theory and focusses on infinite field extensions. The author explains here, among other things, transcendental extensions, separable and p -radical extensions, derivations, and the elements of infinite Galois theory. In addition there are concise accounts of some topics that are important in higher commutative ring theory and algebraic geometry, besides being of independent interest: interactions of subfields, linearly disjoint extensions (and the theorem of F. K. Schmidt), Galois descent, and some Kummer theory.

The entire text is profusely supported by worked examples. Each section in the book comes with a plentiful supply of carefully selected exercises (together with occasional hints), and there are also numerous historical remarks and hints for further reading. Altogether, this textbook breathes once more the author's rich teaching experience and his masterly skill as a textbook writer. His classic three-volume text "Algebra" seems to have undergone remarkable improvements and updates. Already this first volume of its new version appears even more user-friendly, fully ripened and functional than the earlier version. On the other hand, the author has kept his well-tryed methodical principle of combining old and new viewpoints in algebra in a natural way, which makes his textbooks so unique, matchless and timelessly valuable. The mathematical community may look forward to the forthcoming second volume.

Reviewer: [Werner Kleinert \(Berlin\)](#)

MSC:

- 00A05 Mathematics in general
- 16-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to associative rings and algebras
- 13-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to commutative algebra
- 11Exx Forms and linear algebraic groups
- 12-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to field theory
- 20-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to group theory

Cited in 3 Reviews Cited in 34 Documents

Keywords:

group; rings; fields; modules; algebras; graded rings and modules; Noetherian rings; valuation theory; Galois theory; categories; functors; lattices; multilinear algebra; quadratic forms; ordered fields