Dyakonov, Konstantin M.

Differentiation in star-invariant subspaces. I: Boundedness and compactness. (English)

Zbl 1011.47005
J. Funct. Anal. 192, No. 2, 364-386 (2002), erratum 197, No. 2, 576 (2003).

Given two inner functions θ_1, θ_2 on the upper half-plane $\mathbb{C}_+$, let $K^p(\theta_1, \theta_2) = \overline{\theta}_1 H^p \cap \theta_2 \overline{H^p}$, where $H^p = H^p(\mathbb{C}_+)$ is the Hardy space, $p \geq 1$. It is shown that the operator $\frac{d}{dx} : K^p(\theta_1, \theta_2) \to L^p$ is bounded iff $\theta_1', \theta_2' \in L^\infty(\mathbb{R})$; moreover, the norm of the operator is equivalent to $\|\theta_1'\|_\infty + \|\theta_2'\|_\infty$. In addition, the operator is compact iff $\theta_1', \theta_2' \in C_0(\mathbb{R})$.

This implies the following result. Let $\mathcal{R}^p_\Lambda$ be the closed subspace of $L^p(\mathbb{R})$ generated by the rational functions $\{(x-\lambda)^{-j} : 1 \leq j \leq m(\lambda),\ \lambda \in \Lambda\}$, where Λ is a discrete subset of $\mathbb{C} \setminus \mathbb{R}$. Then the operator $\frac{d}{dx} : \mathcal{R}^p_\Lambda \to L^p$ is bounded iff $\mathcal{F}_\Lambda \in L^\infty(\mathbb{R})$, and compact iff $F_\Lambda \in C_0(\mathbb{R})$; here

$$\mathcal{F}_\Lambda(x) = \sum_{\lambda \in \Lambda} m(\lambda) \frac{|\mathrm{Im}\lambda|}{|x-\lambda|^2}, \quad x \in \mathbb{R}.$$

For part II, cf. ibid. 387–409 (2002; Zbl 1011.47006).

Reviewer: Alexandr Yu.Rashkovsky (Khar'kov)

MSC:

47A15	Invariant subspaces of linear operators
30D50	Blaschke products, etc. (MSC2000)
30D55	H^p-classes (MSC2000)

Keywords:

star-invariant subspaces; differentiation operator; Bernstein's inequality; inner functions

Full Text: DOI

References:

[1] Ahern, P.R.; Clark, D.N., Radial limits and invariant subspaces, Amer. J. math., 92, 332-342, (1970) · Zbl 0197.39202
[2] Ahern, P.R.; Clark, D.N., On inner functions with \textit{H}p-derivative, Michigan math. J., 21, 115-127, (1974) · Zbl 0277.30027
[3] A. D. Baranov, Differentiation in de Branges spaces and embedding theorems, in Nonlinear Equations and Mathematical Analysis, J. Math. Sci. (New York)1012000, 2881-2913.
[4] Calderón, A.P., Commutators of singular integral operators, Proc. natl. acad. sci. USA, 53, 1092-1099, (1965) · Zbl 0151.16901
[5] Calderón, A.P., Cauchy integrals on Lipschitz curves and related operators, Proc. natl. acad. sci. USA, 74, 1324-1327, (1977) · Zbl 0373.44003
[6] Cima, J.A.; Ross, W.T., The backward shift on the Hardy space, (2000), Amer. Math. Soc Providence · Zbl 0952.47029
[7] Cohn, W.S., Radial limits and star invariant subspaces of bounded Mean oscillation, Amer. J. math., 108, 719-749, (1986) · Zbl 0607.30034
[8] Douglas, R.G.; Sarason, D.E., A class of Toeplitz operators, Indiana univ. math. J., 20, 891-895, (1971) · Zbl 0199.19103
[9] Douglas, R.G.; Shapiro, H.S.; Shields, A.L., Cyclic vectors and invariant subspaces for the backward shift operator, Ann. inst. Fourier (Grenoble), 20, 37-76, (1970) · Zbl 0186.45302
[10] Dyakonov, K.M., Entire functions of exponential type and model subspaces in \textit{H}p, Zap. nauchn. sem. leningrad otdel. mat. inst. Steklov. (LOMI), 190, 81-100, (1991) · Zbl 0788.30024
[11] Dyakonov, K.M., Smooth functions in the range of a Hankel operator, Indiana univ. math. J., 43, 805-838, (1994) · Zbl 0821.30026
[12] Dyakonov, K.M., Continuous and compact embeddings between star-invariant subspaces, Oper. theory adv. appl., 113, 65-76, (2000) · Zbl 0969.30018
[13] Garnett, J.B., Bounded analytic functions, (1981), Academic Press New York · Zbl 0469.30024

[14] Havin, V.; Jöricke, B., The uncertainty principle in harmonic analysis, (1994), Springer-Verlag Berlin · Zbl 0827.42001

[15] Koosis, P., Interior compact spaces of functions on a half-line, Comm. pure appl. math., 10, 583-615, (1957) · Zbl 0080.31801

[16] Nikol'skiĭ, N.K., Treatise on the shift operator, (1986), Springer-Verlag Berlin

[17] Stein, E.M., Harmonic analysis: real-variable methods, orthogonality, and oscillatory integrals, (1993), Princeton Univ. Press Princeton · Zbl 0821.42001

[18] Yosida, K., Functional analysis, (1965), Springer-Verlag Berlin · Zbl 0126.11504