zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Da Prato, Giuseppe; Röckner, Michael

Singular dissipative stochastic equations in Hilbert spaces. (English) Zbl 1036.47029

Probab. Theory Relat. Fields 124, No. 2, 261-303 (2002); erratum ibid. 143, No. 3-4, 659-664 (2009).

The authors construct weak solutions to SDEs of the form

$$dX = (AX + F_0(X))\,dt + \sqrt{C}\,dW_t, \qquad X(0) = x \in H$$

on a Hilbert space H. In the equation, W_t is a cylindrical Wiener process, C is a positive definite, bounded self adjoint linear operator on H, A is the generator of a strongly continuous semigroup on H, and $F_0(x) := y_0$ where $y_0 \in F(x)$, $|y_0| = \min_{y\in F(x)} |y|$ and F is a maximally dissipative map from H to its power set.

The solution is constructed in two steps: first, the authors solve the corresponding Kolmogorov equations in a suitable L^2-space and construct thus a strong Markov diffusion semigroup. In a second step, it is then shown that the Markov semigroups have a suitable (strong) Fellerian modification which allows to get a proper conservative diffusion process for the solutions of the single starting points. The last two sections deal with uniqueness of the solution and applications, in particular gradient systems and reaction-diffusion equations.

Reviewer: René L. Schilling (Brighton)

MSC:

47D07	Markov semigroups and applications to diffusion processes
35K90	Abstract parabolic equations
60H15	Stochastic partial differential equations (aspects of stochastic analysis)
47B44	Linear accretive operators, dissipative operators, etc.

Keywords:

stochastic differential equations on a Hilbert space; infinite-dimensional analysis; diffusion operator; martingale problem; C_0-semigroup; dissipativity; infinitesimally invariant measure; Feller property; Kolmogorov equations; Kolmogorov's continuity criterion; gradient system; reaction-diffusion equation

Full Text: DOI