**Takahasi, Sin-Ei; Hatori, Osamu; Watanabe, Keiichi; Miura, Takeshi**
**A note on a class of Banach algebra-valued polynomials.** (English) Zbl 1024.46013
Int. J. Math. Math. Sci. 32, No. 3, 189-192 (2002).

Let $E$ be a Banach space and let $F$ be a (unital) Banach algebra. The authors consider two classes of $n$-homogeneous polynomials of finite type from $E$ into $F$, the common finite polynomials

$$P_f(^nE;F)=\{f_1^n\otimes b_1+\cdots+f_k^n\otimes b_k : f_j\in E',\ b_j\in F,\ 1\le j\le k,\ k\in\mathbb{N}\}$$
$$\text{and} P_{\text{fin}}(^nE;F)=\{T_1^n+\cdots+T_k^n : T_j\in L(E;F),\ 1\le j\le k,\ k\in\mathbb{N}\},$$

where $T_j^n(x):=(T_j(x))^n$.

The main results are the following.

Let $F$ be a Banach algebra. Then the following are equivalent.

(a) $F$ is a finite-dimensional space,

(b) $P_{fin}(^nE;F)\subseteq P_f(^nE;F)$ for every $n\in\mathbb{N}$ and every Banach space $E$,

(c) $P_{fin}(^1E;F)\subseteq P_f(^1E;F)$ for every Banach space $E$.

Using a result from *M. L. Lourenço* and *L. A. Moraes* [Publ. Res. Inst. Math. Sci. 37, 521-529 (2001; Zbl 1092.46032)] this is also true with equality in (b) and (c) for unital Banach algebras $F$.

Lemma 2.1 is a special case of the well-known polarization formula.

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**MSC:**

46G25 (Spaces of) multilinear mappings, polynomials
46H20 Structure, classification of topological algebras
47H60 Multilinear and polynomial operators

**Keywords:**

$n$-homogeneous polynomials of finite type; Banach algebras

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