

**Musiał, Kazimierz**

**Pettis integral.** (English) [Zbl 1043.28010](#)

Pap, E. (ed.), Handbook of measure theory. Vol. I and II. Amsterdam: North-Holland (ISBN 0-444-50263-7/hbk). 531-586 (2002).

This extensive survey covers and summarizes the main facts about the Pettis integral for  $X$ -valued functions when  $X$  is an infinite dimensional Banach space. It is organized as follows: The Introduction starts with a brief history of the initial developments of this integral and describes the topics that are not included in the article, such as the setting of locally convex spaces, the approach via Stone transform and some geometric properties of Banach spaces.

Sections “1. Preliminaries”, “2. Measurable functions”, and “3. Scalar integrals, basic properties” contain the basic facts about measurability and integrability, and the class  $\mathbb{P}(\mu, X)$  of Pettis  $\mu$ -integrable  $X$ -valued functions is defined,  $\mu$  being a finite complete measure.

In “4. Pettis integral”, conditions equivalent to the Pettis integrability are included, as well as its relation with Bourgain property and lifting.

The purpose of Section “5. Limit theorems” is to prove the convergence theorems of Vitali and Lebesgue for the Pettis integral.

“6. The range of the Pettis integral” deals with the question of whether this range is relatively compact, and includes the negative answer given by the example due to Fremlin and Talagrand.

“7. Universal integrability” refers to properties of Radon measures and to decomposition properties of universally measurable functions.

“8. Pettis integral property” (PIP) is devoted to a description of Banach spaces that satisfy different variants of this property, such as the PIP for all measures  $\mu$ , the Lebesgue PIP and the universal PIP.

“9. Weak Radon-Nikodým property and related topics” contains a number of contributions of the author in this topic that combined with some other results allow to give several equivalent descriptions of weak\*-compact sets that have the weak Radon-Nikodým property.

“10. Conditional expectation”, strongly related with the previous section, describes some cases where a function has a conditional expectation with respect to a given  $\sigma$ -algebra.

“11. Differentiation” deals with the weak differentiability property

$$\lim_{h \rightarrow 0} (1/h) \int_h^{t+h} \langle x^*, f(\omega) \rangle d\mu = x^* f(t)$$

for a Pettis integrable function  $f$ , and includes an sketch of the proof of Dilworth and Girardi’s result proving that there exist functions that are nowhere differentiable in this weak sense.

“12. Fubini theorem” contains the recent results by Michalak on this topic.

“13. Spaces of Pettis integrable functions” refer to properties of  $\mathbb{P}(\mu, X)$  as a normed space, including a sketch of the proof of facts such as that it is barreled (Drewnowski, Florencio and Paúl) and ultra-bornological (Díaz et al.).

This is only a brief sketch of the article’s structure. It contains more, and it is very clear and readable . This survey, accompanied by the previous notes of the author *Topics in the theory of Pettis integration* [Rend. Ist. Mat. Univ. Trieste 23, No. 1, 177–262 (1991; [Zbl 0798.46042](#))], and by *M. Talagrand’s memoir Pettis integral and measure theory* [Mem. Am. Math. Soc. 307, 224 p. (1984; [Zbl 0582.46049](#))], is an excellent reference for researchers and beginners in the area.

For the entire collection see [[Zbl 0998.28001](#)].

Reviewer: [Joan Cerdà \(Barcelona\)](#)

**MSC:**

28B05 Vector-valued set functions, measures and integrals

46G10 Vector-valued measures and integration

Cited in **27** Documents

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Pettis integral; weak Radon-Nikodým property; liftings; universal measurability; conditional expectation; differentiation; Fubini theorem