Pascasio, Arlene A.

An inequality in character algebras. (English) Zbl 1014.05076

Discrete Math. 264, No. 1-3, 201-209 (2003).

Summary: We prove the following theorem: Let $\mathcal{A} = \langle A_0, A_1, \ldots, A_d \rangle$ denote a complex character algebra with $d \geq 2$ which is P-polynomial with respect to the ordering $A_0, A_1, \ldots, A_d$ of the distinguished basis. Assume that the structure constants p_{ij}^h are all nonnegative and the Krein parameters q_{ij}^k are all nonnegative. Let θ and θ' denote eigenvalues of A_1, other than the valency $k = k_1$. Then the structure constants $a_1 = p_{11}^1$ and $b_1 = p_{12}^1$ satisfy

$$\left(\theta + \frac{k}{a_1 + 1}\right)\left(\theta' + \frac{k}{a_1 + 1}\right) \geq -\frac{ka_1b_1}{(a_1 + 1)^2}.$$

Let E and F denote the primitive idempotents of $\mathcal{A}$ associated with θ and θ', respectively. Equality holds in the above inequality if and only if the Schur product $E \circ F$ is a scalar multiple of a primitive idempotent of $\mathcal{A}$.

The above theorem extends some results of *A. Jurišić*, *J. Koolen* and *P. Terwilliger* [J. Algebr. Comb. 12, 163-197 (2000; Zbl 0959.05121)], and the present author [J. Algebr. Comb. 10, 47-59 (1999; Zbl 0927.05085)]. These people previously showed the above theorem holds for those character algebras isomorphic to the Bose-Mesner algebra of a distance-regular graph.

MSC:

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Cited in **9** Documents

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character algebra; Bose-Mesner algebra; distance-regular graph; association scheme; Krein parameter

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