Hardcastle, D. M.; Khanin, K.

The d-dimensional Gauss transformation: Strong convergence and Lyapunov exponents.
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Let $\Delta^d = \{(\omega_1, \ldots, \omega_d) \in [0,1]^d : \omega_1 \geq \cdots \geq \omega_d\}$ and define the d-dimensional ordered Jacobi-Perron algorithm (JPA) $T : \Delta^d \to \Delta^d$ by

$$T(\omega_1, \ldots, \omega_d) = \begin{cases} (\{\frac{1}{\omega_1}\}, \frac{\omega_2}{\omega_1}, \ldots, \frac{\omega_d}{\omega_1}) & \text{if } \{\frac{1}{\omega_1}\} > \frac{\omega_2}{\omega_1}; \\ (\frac{\omega_2}{\omega_1}, \ldots, \frac{\omega_j}{\omega_1}, \{\frac{1}{\omega_1}\}, \frac{\omega_{j+1}}{\omega_1}, \ldots, \frac{\omega_d}{\omega_1}) & \text{if } \frac{\omega_j}{\omega_1} > \{\frac{1}{\omega_1}\} > \frac{\omega_{j+1}}{\omega_1}; \\ (\frac{\omega_2}{\omega_1}, \ldots, \frac{\omega_d}{\omega_1}, \{\frac{1}{\omega_1}\}) & \text{if } \frac{\omega_d}{\omega_1} > \{\frac{1}{\omega_1}\}, \end{cases}$$

where $\{x\}$ denotes the fractional part of x. This is also called the Gauss algorithm, and it is equivalent to Brun's algorithm and to the modified JPA.

It is proved that the three-dimensional Gauss algorithm is strongly convergent almost everywhere on X. The proof involves the computer assisted estimation of the largest Lyapunov exponent of a cocycle associated to the algorithm.

Such a numerical scheme was used by *S. Ito, M. Keane* and *M. Ohtsuki* [Ergodic Theory Dyn. Syst. 13, 319-334 (1993; Zbl 0846.28005)] to prove almost everywhere strong convergence of the two-dimensional modified Jacobi-Perron algorithm. The authors discuss the scheme in arbitrary dimension to show how the error terms can be estimated explicitly. By this computer assisted proof, they describe how to reduce the problem to a finite number of calculations.

Reviewer's remark: Numerical results which prove almost everywhere strong convergence of the three-dimensional Gauss algorithm have appeared in the same journal [*D. M. Hardcastle*, Exp. Math. 11, 131-141 (2002; Zbl 1022.11034)].

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MSC:

11J70 Continued fractions and generalizations
11K50 Metric theory of continued fractions

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Multidimensional continued fractions; Gauss transformation; strong convergence; Lyapunov exponents; Brun's algorithm; Jacobi-Perron algorithm

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