Hardcastle, D. M.
The three-dimensional Gauss algorithm is strongly convergent almost everywhere. (English)
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Let $X = \{(x_1, x_2, x_3) \in [0,1]^3 \mid x_1 \geq x_2 \geq x_3\}$ and define the three-dimensional ordered Jacobi-Perron algorithm(JPA) $T : X \to X$ as

$$T(x_1,x_2,x_3) = \begin{cases} (\{\frac{1}{x_1}\}, \frac{x_2}{x_1}, \frac{x_3}{x_1}) & \text{if } \{\frac{1}{x_1}\} > \frac{x_2}{x_1}, \\ (\frac{x_2}{x_1}, \{\frac{1}{x_1}\}, \frac{x_3}{x_1}) & \text{if } \frac{x_2}{x_1} > \{\frac{1}{x_1}\} > \frac{x_3}{x_1}, \\ (\frac{x_2}{x_1}, \frac{x_3}{x_1}, \{\frac{1}{x_1}\}) & \text{if } \frac{x_3}{x_1} > \{\frac{1}{x_1}\}, \end{cases}$$

where $\{x\}$ denotes the fractional part of x. This is also called the Gauss algorithm, and it is equivalent to Brun's algorithm and to the modified JPA. It is proved that the three-dimensional Gauss algorithm is strongly convergent almost everywhere on X. The proof involves the computer assisted estimation of the largest Lyapunov exponent of a cocycle associated to the algorithm.

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MSC:

11J70 Continued fractions and generalizations
11K50 Metric theory of continued fractions

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