Alòs, Elisa; **Mazet, Olivier**; **Nualart, David**

Stochastic calculus with respect to Gaussian processes. (English) Zbl 1015.60047

Ann. Probab. 29, No. 2, 766-801 (2001).

The authors consider a family of Gaussian processes $(B_t)_{t\in\mathbb{R}_+}$ of the form $B_t = \int_0^t K(t,s)dW_s$, where K is a deterministic kernel and $(W_t)_{t\in\mathbb{R}_+}$ is a standard Wiener process. They construct a stochastic calculus with respect to such processes via the stochastic calculus of variations, using the anticipating Skorokhod integral operator with respect to $(W_t)_{t\in\mathbb{R}_+}$, which is denoted by δ. The stochastic integral of an adapted process u with respect to $(B_t)_{t\in\mathbb{R}}$ is defined to be $\delta(K^*u)$, where K^* is the adjoint of the operator with kernel K. Itô and Stratonovich change of variable formulas and Hölder regularity results are proved for indefinite integrals with respect to $(B_t)_{t\in\mathbb{R}}$, for a wide class of deterministic (singular and regular) kernels K. The results apply in particular to fractional Brownian motion with Hurst parameter $H \in (1/4, 1/2)$.

Reviewer: Nicolas Privault (La Rochelle)

MSC:

60H05 Stochastic integrals
60H07 Stochastic calculus of variations and the Malliavin calculus
60G15 Gaussian processes

Keywords:

stochastic integrals; Malliavin calculus; Itô formula; fractional Brownian motion

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References:

[1] Al os, E. and Nualart, D. (1998). An extension of It o's formula for anticipating processes. J. Theoret. Probab. 11 493-514. · Zbl 0914.60018 · doi:10.1023/A:1022692024364

[2] Al os, E., Mazet, O. and Nualart, D. (2000). Stochastic calculus with respect to fractional Brownian motion with Hurst parameter less than 12. Stochastic Process. Appl. 86 121-139. · Zbl 1028.60047 · doi:10.1016/S0304-4149(99)00089-7

[3] Carmona, P. and Coutin, L. (1998). Stochastic integration with respect to fractional Brownian motion. · Zbl 0921.60067 · emis:journals/EJP-ECP/EcpVol3/paper12.abs.html · eudml:119936

[4] Comte, F. and Renault, E. (1996). Long memory continuous time models. J. Econometrics 73 101-149. · Zbl 0856.62104 · doi:10.1016/0304-4076(95)01735-6

[5] Dai, W. and Heyde, C. C. (1996). It o's formula with respect to fractional Brownian motion and its application. J. Appl. Math. Stochastic Anal. 9 439-448. · Zbl 0867.60029 · doi:10.1155/S104895339600038X · eudml:47673

[6] Decreusefond, L. and Üst ünel, A. S. (1998). Stochastic analysis of the fractional Brownian motion. Potential Anal. 10 177-214. · Zbl 0924.60034 · doi:10.1023/A:1008634027843

[7] Decreusefond, L. and Üst ünel, A. S. (1998). Fractional Brownian motion: theory and applications. ESAIM: Proceedings 5 75-86. · Zbl 0914.60019 · doi:10.1051/proc:1998014 · www.edpsciences.org

[8] Duncan, T. E., Hu, Y. and Pasik-Duncan, B. (1998). Stochastic calculus for fractional Brownian motion. I. Theory. · Zbl 0947.60061 · doi:10.1137/S036301299834171X

[9] Feyel, D. and de la Pradelle, A. (1996). Fractional integrals and Brownian processes. Potential Anal. 10 273-288. · Zbl 0944.60045 · doi:10.1023/A:1008630211913

[10] Gaveau, B. and Trauber, P. (1982). L'intégrale stochastic comme opérateur de divergence dans l'espace fonctionnel. J. Funct. Anal. 46 230-238. · Zbl 0488.60068 · doi:10.1016/0022-1236(82)90036-2

[11] Hu, Y. and Øksendal, B. (1999). Fractional white noise calculus and applications to finance.

[12] Kleptsyna, M. L., Kloeden, P. E. and Anh, V. V. (1998). Existence and uniqueness theorems for stochastic differential equations with fractal Brownian motion. Problemy Peredachi Informatsii 34 54-56. · Zbl 0924.60042

[13] Lin, S. J. (1995). Stochastic analysis of fractional Brownian motions. Stochastics Stochastics Rep. 55 121-140. · Zbl 0886.60076

[14] Malliavin, P. (1997). Stochastic Analysis. Springer, New York. · Zbl 0878.60001

[15] Mandelbrot, B. B. and Van Ness, J. W. (1968). Fractional Brownian motions, fractional noises and applications. SIAM Rev. 10 422-437. JSTOR: · Zbl 0179.47801 · doi:10.1137/1010093 · links.jstor.org

[16] Norros, I., Valkeila, E. and Virtamo, J. (1999). An elementary approach to a Girsano formula and other analytical results on fractional Brownian motion. Bernoulli 5 571-587. · Zbl 0955.60034 · doi:10.2307/3318691 ·

[17] Nualart, D. (1995). The Malliavin Calculus and Related Topics. Probab. Appl. 21. · Zbl 0837.60050

[18] Nualart, D. (1998). Analysis on Wiener space and anticipating stochastic calculus. Lecture Notes in Math. 1690 123-227. Springer, New York. · Zbl 0915.60062 · doi:10.1007/BFb0092538

[19] Nualart, D. and Pardoux, E. (1988). Stochastic calculus with anticipating integrands. Probab. Theory Related Fields 78 535-581. · Zbl 0629.60061 · doi:10.1007/BF00353876

[20] Samko, S. G., Kilbas, A. A. and Marichev, O. I. (1993). Fractional Integrals and Derivatives. Gordon and Breach, New York. · Zbl 0818.26003

[21] Skorohod, A. V. (1975). On a generalization of a stochastic integral. Theory Probab. Appl. 20 219-233. · Zbl 0333.60060

[22] Zähle, M. (1998). Integration with respect to fractal functions and stochastic calculus. I. Probab. Theory Related Fields 111 333-374. · Zbl 0918.60037 · doi:10.1007/s004400050171

[23] Zähle, M. (1999). Integration with respect to fractal functions and stochastic calculus. II. · Zbl 0983.60054