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Wilson's Grassmannian and a noncommutative quadric. (English) Zbl 1059.58006

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Introduction: Let the group μ_m of m th roots of unity act on the complex line by multiplication. This gives a μ_m -action on Diff , the algebra of polynomial differential operators on the line. Following *W. Crawley-Boevey* and *M. P. Holland* [Duke Math. J. 92, No. 3, 605-635 (1998; Zbl 0974.16007)], we introduce a multiparameter deformation D_τ of the smash product $\text{Diff}\#\mu_m$. Our main result provides natural bijections between (roughly speaking) the following spaces:

- (1) μ_m -equivariant version of Wilson's adelic Grassmannian of rank r ;
- (2) rank r projective D_τ -modules (with generic trivialization data);
- (3) rank r torsion-free sheaves on a "noncommutative quadric" $\mathbb{P}^1 \times_\tau \mathbb{P}^1$;
- (4) disjoint union of Nakajima quiver varieties for the cyclic quiver with m vertices.

The bijection between (1) and (2) is provided by a version of Riemann-Hilbert correspondence between D -modules and sheaves. The bijections between (2), (3), and (4) were motivated by our previous work [Compos. Math. 134, No. 3, 283-318 (2002; Zbl 1048.14001)]. The resulting bijection between (1) and (4) reduces, in the very special case: $r = 1$ and $\mu_m = \{1\}$, to the partition of (rank 1) adelic Grassmannian into a union of Calogero-Moser spaces discovered by Wilson. This gives, in particular, a natural and purely algebraic approach to *G. Wilson's* result [Invent. Math. 133, No. 1, 1-41 (1998; Zbl 0906.35089)].

MSC:

- 58B25** Group structures and generalizations on infinite-dimensional manifolds
- 16S32** Rings of differential operators (associative algebraic aspects)
- 32C38** Sheaves of differential operators and their modules, D -modules
- 14A22** Noncommutative algebraic geometry
- 14M15** Grassmannians, Schubert varieties, flag manifolds

Cited in 11 Documents

Keywords:

algebras of polynomial differential operators; multiparameter deformations; smash products; adelic Grassmannians; projective D -modules; sheaves; quadrics; Riemann-Hilbert correspondence; quiver varieties; Calogero-Moser spaces

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