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Optimal linear Bayes and empirical Bayes estimation and prediction of the finite population mean. (English) Zbl 1015.62003

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Summary: We investigate the estimation problem of the population mean of a finite population. Both point and interval estimators are of interest from Bayes and empirical Bayes points of view. Empirical Bayes analysis is concerned with the 'current' population mean, say γ_m, when the sample data are available from other similar $(m-1)$ finite populations, $\mathbf{Y}_1, \ldots, \mathbf{Y}_{m-1}$, as well as the data from the current population, $\mathbf{Y}_m$, where $\mathbf{Y}_i = (Y_{i1}, \ldots, Y_{in_i})$, $i = 1, \ldots, m$.

Previous results on inference of γ_m have assumed either a normal model or a posterior linearity condition in making Bayes inference which is the kernel of the empirical Bayes problem. They resulted in examination of linear estimators of the sample mean $\overline{Y}_m = n_m^{-1} \sum_{j=1}^{n_m} Y_{mj}$. We propose to investigate a generalizing idea which generates optimal linear Bayes estimators of γ_m as functions of $\overline{Y}_m$. We develop optimal linear Bayes estimators of γ_m under two Bayesian models. They are optimal in the sense of minimizing the mean squared error with respect to the underlying models. The corresponding empirical Bayes analogues are obtained by replacing the unknown hyperparameters by their respective consistent estimates as usual. The asymptotic optimality criterion is employed in order to measure the goodness of the proposed empirical Bayes estimators. Very promising Bayes and empirical Bayes two-sided confidence intervals and predictors of γ_m are also discussed. A Monte Carlo study is conducted to evaluate the performance of the proposed estimators.

MSC:

62D05 Sampling theory, sample surveys
62C12 Empirical decision procedures; empirical Bayes procedures
62F15 Bayesian inference

Keywords:

finite populations; Bayes estimators; linear estimators; empirical Bayes; asymptotic optimality

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