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**Subexponential decay of correlations.** (English) Zbl 1042.37005

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Let $(X,\mathcal{B},m,T)$ be a dynamical system, and denote by $\widehat{T}$ the transfer operator (Perron-Frobenius operator) associated with the dynamical system. The first assertion is the following renewal equation.

Proposition 1. Let the dyamical system be conservative and nonsingular. Assume that $A\in\mathcal{B}$ has finite positive measure. Let $T_nf=1_A\widehat{T}^n(f1_A)$ and $R_nf=1_A\widehat{T}^n(f1_{[\phi_A=n]})$, where $\phi_A(x)$ is the first return time to $A$ for $x\in A$. Then for any $|z|<1$

$$T(z)=(I-R(z))^{-1},\quad R(z)=\sum_{n=1}^{\infty}a^nR_n,\quad T(z)=\sum_{n=0}^{\infty}z^nT_n.$$

The main result of this article is:

Theorem 1. Let $T_n$ be bounded linear operators on a Banach space $\mathcal{L}$ such that $T(z)=\sum_{n=0}^{\infty}z^nT_n$ converges in $\mathrm{Hom}(\mathcal{L},\mathcal{L})$ for every $|z|<1$. Assume the

Renewal equation: for every $|z|<1$, $T(z)=(I-R(z))^{-1}$ where $R(z)=\sum_{n\geq 1}z^nR_n$, $R_n\in\mathrm{Hom}(\mathcal{L},\mathcal{L})$ and $\sum\|R_n\|<\infty$.

Spectral gap: the spectum of $R(1)$ consists of an isolated simple eigenvalue at 1 and a compact subset in $|z|<1$.

Aperiodicity: the spectral radius of $R(z)$ is strictly less than one for all $|z|\leq 1$ and $z\neq 1$.

Let $P$ be the eigenprojection of $R(1)$ at 1.

If $\sum_{k>n}\|R_k\|=O(1/n^{\beta})$ for some $\beta>2$ and $PR'(1)\neq 0$, then for all $n$

$$T_n=\frac{1}{\mu}P+\frac{1}{\mu^2}\sum_{k=n+1}^{\infty}P_k+E_n$$

where $\mu$ is given by $PR'(1)P=\mu P$, $P_n=\sum_{l>n}PR_lP$, and $E_n\in\mathrm{Hom}(\mathcal{L},\mathcal{L})$ satisfy $\|E_n\|=O(1/n^{[\beta]})$.

Theorem 2 gives lower bounds for the correlation functions $s$:

For a Markov partition $\alpha$, let $[a_,\dots,a_{n-1}]$ be the cylinder generated by $a_1,\dots,a_{n-1}\in\alpha$. Let

$$v_n(\phi)=\sup\{|\phi(x)-\phi(y)|:x,y\in[a_0,\dots,a_{n-1}],a_i\in\alpha\}.$$

The dynamical system is called $(T,\alpha)$-summable if $\sum_{n\geq 2}v_n(\phi)<\infty$, and $(T,\alpha)$-locally Hölder continuous if there exists $A>0$, $\theta\in(0,1)$ such that $v_n(\phi)<A\theta^{\overline{n}}$ for all $n$. Let $T_a$ be a induced transformation on $a\in\alpha$, and the Markov partition induced on $a$ is denoted by $\alpha_a$. Moreover

$$g_m=\frac{dm}{dm\circ T},$$
$$s(x,y)=\sup\{n\geq 0:x,y\in[b_0,\dots,b_{n-1}],b_i\in\alpha_a\},$$
$$D_af=\sup|f(x)-f(y)|/\theta^{s(x,y)}.$$

Define

$$\|f\|_{\mathcal{L}}=\|f\|_{\infty}+D_af.$$

Then Theorem 2. For a measure-preserving, irreducible Markov dynamical system, assume that $\log g_{m_a}$ has $(T_a,\alpha_a)$-locally Hölder continuous version for some $a\in\alpha$. If g.c.d. $\{\phi_a(x)-\phi-a(y):x,y\in\cup\alpha_a\}=1$ and $m[\phi_a>n]=O(1/n^{\beta})$ where $\beta>2$, then there exists $\theta\in(0,1)$ $C>0$ such that for any $f,g$ integrable supported inside $[a]$,

$$\left|\mathrm{Cor}(f, g\circ T^n) - \left(\sum_{k=n+1}^{\infty} m[\phi_a > k]\right)\int f \int g\right| \leq Cn^{-[\beta]}\|g\|_\infty \|f\|_{\mathcal{L}}.$$

Two examples are given and correlations of them are calculated. One is an extension of the Mannville-Pomeau map, and the other are LS Young towers.

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**MSC:**

37A30 Ergodic theorems, spectral theory, Markov operators
28D05 Measure-preserving transformations
47A35 Ergodic theory of linear operators
37C30 Functional analytic techniques in dynamical systems; zeta functions, (Ruelle-Frobenius) transfer operators, etc.



**Keywords:**

transfer operator; renewal equation; measure-preserving irreducible Markov dynamical system

**Full Text:** DOI