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Back and forth error compensation and correction methods for removing errors induced by uneven gradients of the level set function. (English) Zbl 1031.65110

J. Comput. Phys. 190, No. 1, 311-324 (2003).

The paper considers the reduction of numerical errors in the solution of time dependent linear advection equations which occur, e.g., in level set methods. Two schemes for error reduction are proposed. Both schemes, the back error compensation method as well as the forth error compensation method perform as a first step the solution of the equation with a basic scheme from time t_n to t_{n+1} and as second step a backward solution with the same scheme from t_{n+1} to t_n.

Ideally, the solution on t_n should be recovered. However, due to numerical errors this is in general not the case. Based on the error, the backward error compensation error method defines as third step a modified solution in t_n. With this modified solution, the final solution in t_{n+1} is computed. In the forward error compensation method, the third step is again a solve from t_n to t_{n+1} based on the solution obtained with the second step.

The csation of the error is now performed using the difference of the solution after the first and third step. For the case of applying the backward error compensation method to an ordinary differential equation, it is proven that this method improves the order of accuracy for certain schemes. In addition, a simple stability result is proven for the one-dimensional translation equation.

Numerical tests using as basic scheme an upwind scheme of first order are presented, in particular for Zalesak's problem. They show a considerable improvement of accuracy using both error compensation approaches in comparison to using the basic scheme without error compensation. Finally, there is a short comparison of the presented approach to other methods of error reduction from the literature.

Reviewer: Volker John (Magdeburg)

MSC:

65M60	Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs
65M12	Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs
65M15	Error bounds for initial value and initial-boundary value problems involving PDEs
35L45	Initial value problems for first-order hyperbolic systems

Keywords:

flux corrected transport; front tracking; level set method; numerical examples; comparison of methods; linear advection equations; backward error compensation method; error reduction; forward error compensation method; stability; translation equation; Zalesak's problem

Software:

SHASTA

Full Text: DOI

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