Descombes, Stéphane; **Schatzman, Michelle**

Strang's formula for holomorphic semi-groups. (English) Zbl 1030.35095

J. Math. Pures Appl., IX. Sér. 81, No. 1, 93-114 (2002).

In the finite-dimensional case G. Strang proved the following approximation formula

$$\|e^{-t(A+B)} - e^{-tA/2}e^{-tB}e^{-tA/2}\| = O(t^3).$$

The authors prove generalizations of this formula to the case of infinite dimensional Banach and unbounded generators A, B. Detailed analysis of Schrödinger operators ($A = -\Delta, B = V(x)$) in $L^p(\mathbb{R}^d)$, matrix Schrödinger operator and the pair of elliptic second-order operators ($A = \frac{d}{dx}a\frac{d}{dx}, A = \frac{d}{dx}b\frac{d}{dx}$) in $L^2(\mathbb{R}^d)$ is realized.

Reviewer: Michael Perelmuter (Kyïv)

MSC:

35K15 Initial value problems for second-order parabolic equations
47D06 One-parameter semigroups and linear evolution equations
35A35 Theoretical approximation in context of PDEs

Keywords:

Schrödinger operators; matrix Schrödinger operator; pair of elliptic second-order operators

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