Beukers, F.; **Smyth, C. J.**

Cyclotomic points on curves. (English) Zbl 1029.11009

Bennett, M. A. (ed.) et al., Number theory for the millennium I. Proceedings of the millennial conference on number theory, Urbana-Champaign, IL, USA, May 21-26, 2000. Natick, MA: A K Peters. 67-85 (2002).

Let $f(x,y)$ be a Laurent polynomial with complex coefficients, and let $V(f)$ be the area of its Newton polytope. A pair (a,b) is said to be a cyclotomic point if both a and b are roots of unity. Suppose that f is such that there are only finitely many cyclotomic points (a,b) for which $f(a,b)=0$. The authors prove that then on the curve $f(x,y)=0$ there are at most $22V(f)$ cyclotomic points. They give an example of

$$f(x,y) = xy + 1/xy + x + 1/x + y + 1/y + 1$$

which shows that the constant 22 cannot be replaced by a constant smaller than 16. An infinite family of polynomials for which the constant is approximately 10 is also constructed. Finally, the authors give a sharp version of their upper bound for the number of cyclotomic points on a curve. Since one needs extra work to compute this sharp bound, the bound $22V(f)$ seems more practical for applications. In the paper, the authors also give an algorithm for finding the cyclotomic part of a polynomial in one variable and the literature where one can find much more general (but not so sharp) results.

For the entire collection see [Zbl 1002.00005].

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MSC:

11C08 Polynomials in number theory
11R09 Polynomials (irreducibility, etc.)

Keywords:

roots of unity; polynomials; Newton polytope