Liao, Shijun

Beyond perturbation. Introduction to the homotopy analysis method. (English)

Zbl 1051.76001

CRC Series: Modern Mechanics and Mathematics. Boca Raton, FL: Chapman and Hall/CRC (ISBN 1-58488-407-X/hbk; 978-0-203-49116-4/ebook). xii, 322 p. (2004).

The author develops the homotopy analysis method (HAM) for general nonlinear problems. The basic ideas of HAM are presented in introduction (Ch. 1) and explained on simple nonlinear ordinary differential equation (Ch. 2). Here the author shows that the usual perturbation method, Lyapunov's artificial small parameter method, Adomian's decomposition method and δ-expansion method can be considered as particular cases of HAM. In Ch. 3 a systematic description of HAM for general nonlinear problems is given. Here fundamental rules, such as the rule of solution expression, the rule of coefficient erodicity and the rule of solution existence are presented, which considerably simplify the application of HAM. Control of convergence regions and rates together with homotopy-Padé technique are considered. Ch. 4 contains in abstract form the relationships between the homotopy analysis method and the above-mentioned other nonperturbation techniques. In Ch. 5 some discussions about the advantages and limitations of HAM are presented, and some open questions are pointed out.

The larger second part contains various applications. These are the Duffing oscillator (Ch. 6) together with the question of multiple solutions of a nonlinear problem (Ch. 7), nonlinear eigenvalue problems (Ch. 8), Thomas-Fermi atom model (Ch. 9), and Volterra model for the population growth described by a nonlinear integro-differential equation (Ch. 10). In Chs. 11 and 12 free oscillations of a general conservative system of the second order with odd nonlinearity and with quadratic nonlinearity are considered. Ch. 13 solves the limit cycle problem for one-dimensional nonlinear dynamical systems governed by differential equation $\ddot{u}(t) = f(u, \dot{u})$. The rest part of the book is devoted to HAM applications to hydrodynamical problems: Blasius viscous flow (Ch. 14), boundary-layer flows with exponential property (Ch. 15), boundary-layer flows with algebraic property (Ch. 16), von Kármán swirling viscous flow (Ch. 17), and the problem on nonlinear progressive waves in deep water (Ch. 18).

This monograph offers the opportunity to explore the details of the valuable new approach both in the theory and on many interesting examples. It will be useful to specialists working in applied nonlinear analysis.

Reviewer: Boris V. Loginov (Ul'yanovsk)

MSC:

76-02	Research exposition (monographs, survey articles) pertaining to fluid mechanics
70-02	Research exposition (monographs, survey articles) pertaining to mechanics of particles and systems
76M45	Asymptotic methods, singular perturbations applied to problems in fluid mechanics
70K60	General perturbation schemes for nonlinear problems in mechanics

Keywords:

Duffing oscillator; nonlinear eigenvalue problems; free oscillations; boundary-layer flows; nonlinear waves

Full Text: DOI