Lando, S. K.; **Zvonkin, A. K.**
Graphs on surfaces and their applications. Appendix by Don B. Zagier. (English)
Zbl 1040.05001
Encyclopaedia of Mathematical Sciences 141. Low-Dimensional Topology 2. Berlin: Springer (ISBN 3-540-00203-0/hbk). xv, 455 p. (2004).

The goal of this book is to explain the interrelations between three distinct ways to consider an embedded graph: as a topological object, as a sequence of permutations, as a way of representing a ramified covering of the sphere by a compact two-dimensional manifold.

Chapter 1 introduces the objects of study, namely constellations (finite sequences of permutations), ramified coverings of the sphere, embedded graphs in a variety of forms, and Riemann surfaces.

Chapter 2 is concerned with dessins d'enfants and the combinatorial and geometric consequences of Belyi's theorem which asserts the faithfulness of a certain group action on maps.

In Chapter 3 the subject of matrix integrals in map enumeration is introduced. The main emphasis here is the interpretation of the matrix integrals in combinatorial terms as well as the encoding of a combinatorial problem in terms of a matrix integral.

The method of matrix integrals is a tool to compute the Euler characteristic of moduli spaces of complex algebraic curves, which are the subject of Chapter 4.

A general study of meromorphic functions is provided in Chapter 5. Enumerative questions are related to algebraic geometry and singularity theory via the Lyashko-Looijenga mapping, while the flexible classification of meromorphic functions is related to braid group action on constellations.

Chapter 6 explains the structure of the Hopf algebra on chord diagrams interpreted as one vertex maps.

An appendix by Don Zaiger provides the reader with a short course on representation and character theory of finite groups, with applications to the enumeration of constellations.

The authors introduce their objects of study, namely graphs and maps, with great care and detail. A wealth of applications is pointed out, but of course these applications are sometimes introduced with minimal or no explanation. The Appendix provides some elegant and concise proofs of results used in the main body of the book and the bibliography contains 313 entries. Well-chosen examples and strategically placed exercises support the reader in gaining an understanding of graphs on surfaces and their fascinating applications.

Reviewer: Brigitte Servatius (Worcester)

MSC:

05-02	Research exposition (monographs, survey articles) pertaining to combinatorics	Cited in **184** Documents
05C10	Planar graphs; geometric and topological aspects of graph theory	
05C30	Enumeration in graph theory	
14H55	Riemann surfaces; Weierstrass points; gap sequences	
15B52	Random matrices (algebraic aspects)	
20Bxx	Permutation groups	
20F36	Braid groups; Artin groups	
30Fxx	Riemann surfaces	
32G15	Moduli of Riemann surfaces, Teichmüller theory (complex-analytic aspects in several variables)	
57M15	Relations of low-dimensional topology with graph theory	
57M12	Low-dimensional topology of special (e.g., branched) coverings	
57M27	Invariants of knots and 3-manifolds (MSC2010)	
81T18	Feynman diagrams	
81T40	Two-dimensional field theories, conformal field theories, etc. in quantum mechanics	

Keywords: