Bleher, Pavel; **Eynard, Bertrand**
Double scaling limit in random matrix models and a nonlinear hierarchy of differential equations. (English) [Zbl 1053.15017]
J. Phys. A, Math. Gen. 36, No. 12, 3085-3105 (2003).

The authors consider the random matrix ensemble

$$d\mu_N(M) = Z_N^{-1} \exp\left(-NTr\,V(M)\right)\,dM$$

on the space of Hermitian $N \times N$ matrices M, where V is a polynomial. In the limit $N \to \infty$, the distribution of eigenvalues is known to be of the form

$$d\nu_\infty(x) \propto h(x) \sqrt{\prod_{j=1}^{q}(x-a_j)(b_j-x)}\,\mathbf{1}_{\{x \in \cup_{j=1}^{q}[a_j,b_j]\}}.$$

$d\nu_\infty(x)$ is said to be regular (otherwise singular) if $h(x) \neq 0$ for $x \in \bigcup_{j=1}^{q}[a_j,b_j]$. If $d\nu_\infty(x)$ is singular, the corresponding polynomial $V(x)$ is said to be critical. In particular, the authors consider the case

$$d\nu_\infty(x) \propto (x-c)^{2m}\sqrt{4-x^2}, \quad m \geq 1.$$

To study the critical behavior in the vicinity of a critical polynomial $V(x)$, one considers a parametric family $V(x;t)$ such that $V(x;t_c) = V(x)$ for some t_c and studies the limit $t \to t_c$. As a first result, in the particular case $m = 1$ it is shown in the paper that the free energy has a third order phase transition at t_c. The other problem is to analyze the asymptotics if the recurrence coefficients defining the orthogonal polynomials with respect to the measure $\exp(-NV(x))$. The relevant limit is the one where n (the order of the polynomial) and N diverge with $n/N \to t/t_c$. The "double scaling limit" consists in letting $n/N \to 1$ with a suitable scaling for $n-N$. In this regime, the authors formulate a scaling Ansatz for the recurrence coefficients, which is consistent with the known asymptotics for $t < t_c$. Some consequences of this Ansatz are then discussed.

Reviewer: Fabio L. Toninelli (Lyon)

MSC:

15B52	Random matrices (algebraic aspects)
82B44	Disordered systems (random Ising models, random Schrödinger operators, etc.) in equilibrium statistical mechanics
82B41	Random walks, random surfaces, lattice animals, etc. in equilibrium statistical mechanics
34M30	Asymptotics and summation methods for ordinary differential equations in the complex domain
37K10	Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)

Cited in **12** Documents

Keywords:

random matrices; double scaling limit; orthogonal polynomials; distribution of eigenvalues; phase transition

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