**Parrilo, Pablo A.**

**Semidefinite programming relaxations for semialgebraic problems.** (English) Zbl 1043.14018
Math. Program. 96, No. 2 (B), 293-320 (2003).

This particularly well written paper can be viewed as a summary of the author's PhD manuscript published in 2000. The paper does not really contain any new theoretical result. Its merit consists of linking in a transparent way various branches of algebra, applied mathematics and engineering, with the help of a set of carefully chosen illustrative numerical examples. The main objective is to convince the reader that many computational problems traditionally deemed as difficult can be approached in a unified way using convex relaxations and semidefinite programming (SDP).

The problems considered in this paper consist in checking whether a finite set of multivariate polynomial equalities and inequalities is empty or not. It is recalled that emptiness of such semialgebraic sets can be proved via the Positivstellensatz (a result of real algebraic geometry) in terms of a polynomial identity. The degree of this polynomial can be very high (triply exponential in the problem dimension). Moreover, part of this polynomial is subject to non-negativity constraints. For these two reasons, finding this polynomial certificate is generally computationally difficult.

The main ideas are then (1) to restrict the search to polynomial certificates of bounded, and possibly low degrees; (2) to relax the non-negativity constraint with a sufficient sum-of-squares (SOS) constraint. Finding the polynomial certificate then boils down to solving a convex SDP optimization problem. With the help of numerical examples, it is shown that this approach can prove particularly useful in practice.

Reviewer: Didier Henrion (Toulouse)

**MSC:**

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|---|---|
| 14P10 | Semialgebraic sets and related spaces |
| 13J30 | Real algebra |
| 90C10 | Integer programming |
| 90C22 | Semidefinite programming |



**Keywords:**

real algebraic geometry; semidefinite programming; multivariate polynomials

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