

**Savin, A. Yu.; Sternin, B. Yu.****Subspaces defined by pseudodifferential projections, and some of their applications.** (English. Russian original) [Zbl 1055.58012](#)

Dokl. Math. 61, No. 2, 235-238 (2000); translation from Dokl. Akad. Nauk, Ross. Akad. Nauk 371, No. 4, 448-451 (2000).

This nicely written short paper reviews the results of the authors from [Sb. Math. 191, No. 8, 1191–1213 (2000; [Zbl 0981.58018](#))] and [Sb. Math. 190, No. 8, 1195–1228 (1999; [Zbl 0963.58008](#))]. The subspaces from the title are images in  $C^\infty(M, E)$  of pseudodifferential projections of order 0 over a closed manifold  $M$  which are “admissible”, in the sense that the parity of their symbol with respect to the antipodal map is opposite to the parity of  $\dim M$ . There exists a map  $d$  from the semigroup of homotopy classes of such projections into  $\mathbb{Z}[1/2]$  with the following property: If  $A$  is an elliptic pseudodifferential operator with “parity” opposite to  $\dim M$ , and  $L_+(A)$  is the image of the spectral projection coming from the nonnegative eigenvalues of  $A$ , then  $d(L_+(A))$  coincides with the eta invariant of  $A$ . This implies that  $\eta(A)$  belongs to  $\mathbb{Z}[1/2]$ , which answers positively a conjecture of *P. Gilkey* [Adv. Math. 58, 243–284 (1985; [Zbl 0602.58041](#))]. The functional  $d$  appears also in an index formula for elliptic operators acting on subspaces as above, as well as for the index of certain boundary-value problems.

Reviewer: [Sergiu Moroianu \(București\)](#)**MSC:**

- [58J40](#) Pseudodifferential and Fourier integral operators on manifolds  
[58J28](#) Eta-invariants, Chern-Simons invariants

**Keywords:**[Eta invariant](#); [index of elliptic operators](#); [boundary-value problems](#)