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Exterior stationary Navier-Stokes flows in 3D with non-zero velocity at infinity: Approximation by flows in bounded domains. (English) Zbl 1050.35067

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The authors consider a stationary incompressible Navier-Stokes equation in an exterior domain $\overline{\Omega}^c$, where Ω is an open and bounded subset of $\mathbb{R}^3$ with Lipschitz boundary and such that Ω^c is connected. The problem is rewritten as $-\Delta u + \tau D_1 u + \widetilde{\tau}(u \cdot \nabla u)u + \nabla \pi = f$ in $\overline{\Omega}^c$, $\operatorname{div}(u) = 0$ in $\overline{\Omega}^c$. The nonhomogeneous Dirichlet boundary conditions $u = b$ are imposed on $\partial\Omega$ and the solution u is supposed to satisfy $|u(x)| =_{|x|\to\infty} O(|x|^{-1})$. f (resp. b) is supposed to belong to $L^{6/5}(\mathbb{R}^3)^3$ (resp. to $H^{1/2}(\partial\Omega)^3$) and satisfies $|f(x)| \leq \gamma |x|^{-\sigma}$ for some $\sigma \in (4,\infty)$. The purpose of the paper is to prove that the solution of this exterior Navier-Stokes problem can be approximated by the solution of an exterior Navier-Stokes problem but posed in a bounded domain $\Omega_R = B_R \setminus \overline{\Omega}$, where B_R is the ball of $\mathbb{R}^3$ centered at the origin and of radius R. The norms of f and b in their respective spaces are supposed to be controlled in terms of τ and $\widetilde{\tau} \in [0,\tau]$. In order to prove the existence of a solution of this exterior problem, the authors introduce six positive constants $S < S_1 < \dots < S_5$. They prove the existence of a solution of this problem $(u,\pi) \in W^{2,6/5}_{\text{loc}}(\overline{\Omega}^c)^3 \times W^{1,6/5}_{\text{loc}}(\overline{\Omega}^c) \cap L^2(\overline{\Omega}^c)$ and an estimate which involves some norms of either u or ∇u in the different annuli associated to the constants S_1, ..., S_5. The main difficulty is to find the correct way to evaluate the difference between the solution of the original problem and that posed in the bounded exterior domain Ω_R. The authors first define a variational formulation associated to the bounded exterior problem. They prove the existence of a solution of this variational problem in $H^1(\Omega_R)^3 \times L^2(\Omega_R)$. The main result of the paper asserts that the norm in $H^1(\Omega_R)^3$ of the difference of the two solutions is bounded by some expression which involves τ and $R^{-1/2}$ for every R in $[S_5,+\infty)$. The norm of the difference of the traces of the solutions on ∂B_R multiplied by $(R^{-1}+\tau)^{1/2}$ is also bounded by the same expression. The proof of this result is mainly based on the integral representation of the fundamental solution of Oseen's problem. Thus doing, they prove an estimate of the gradient of the velocity of this Oseen's problem, estimate which is a correction of the one indicated by the same authors in a previous paper.

Reviewer: Alain Brillard (Mulhouse)

MSC:

35Q30	Navier-Stokes equations
76D05	Navier-Stokes equations for incompressible viscous fluids
76D07	Stokes and related (Oseen, etc.) flows
35J55	Systems of elliptic equations, boundary value problems (MSC2000)

Keywords:

stationary incompressible Navier-Stokes flows; exterior domains; approximation; truncation error

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