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Real Paley-Wiener theorems for the inverse Fourier transform on a Riemannian symmetric space. (English) Zbl 1049.43004

Pac. J. Math. 213, No. 1, 1-13 (2004).

Let G be a noncompact semisimple Lie group and K a maximal compact subgroup of G. *S. Helgason* [Geometric analysis on symmetric spaces (Mathematical Surveys and Monographs 39, Am. Math. Soc., Providence, Rhode Island) (1994; Zbl 0809.53057)] gave a Paley-Wiener theorem for the Fourier transform, which characterizes the image of the compactly supported, infinitely differentiable functions on $X = G/K$ in terms of holomorphic extensions and their growth at infinity, analogous to the classical case. The question addressed by the present paper is what type of Paley-Wiener type theorem is available for the inverse Fourier transform?

When restricted to K-invariant functions, the Fourier transform on X reduces to the spherical transform on G. For the complex, rank one case, *A. Pasquale* [Pac. J. Math. 193, No. 1, 143–176 (2000; Zbl 1014.22010)] has proved a Paley-Wiener theorem for the inverse spherical transform. The present author proves a real Paley-Wiener theorem for the inverse Fourier transform for general Riemannian symmetric spaces. Briefly, it is that for smooth f in $L^2(X)$ to have compactly supported Fourier transform it must satisfy $\lim \|\Delta^n f\|_2^{1/2n} < \infty$ where Δ is the Laplace-Beltrami operator.

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