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On differentiability of SRB states for partially hyperbolic systems. (English) Zbl 1059.37021

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It is well known that the question of differentiability of SRB measures (and u-Gibbs states) plays an important role in averaging, rigidity theory and statistical physics, but not much is known beyond the uniformly hyperbolic case. The paper is devoted to the question of differentiability of u-Gibbs states for partially hyperbolic systems and presents a new method to prove differentiability. This method is outlined on the example of abelian Anosov actions (partially hyperbolic systems whose central directions are spanned by the symmetry (abelian) group of the system).

To be more specific, the paper considers a one-parameter family of diffeomorphisms f_ε such that f_0 is an Anosov element in a standard abelian Anosov action having sufficiently strong mixing properties (f_0 is rapidly mixing). Let ν_ε be any u-Gibbs state for f_ε. The main result states that for any C^∞ function A, the map $\varepsilon \to \nu_\varepsilon(A)$ is differentiable at $\varepsilon = 0$. This implies that the difference of Birkhoff averages of the perturbed and unperturbed systems is proportional to ε. This result is applied to show that a generic perturbation of the time one map of geodesic flow on the unit tangent bundle over a surface of negative curvature has a unique SRB measure with good statistical properties.

The main difference between the approach used in the paper and the previous work on differentiability is that there are no any assumptions on the dynamics of the perturbed system that extends greatly the range of applicability of the method. An important point is that the approach does not depend on a particular structure of abelian Anosov actions, so it seems possible to extend results obtained to a more general setting.

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MSC:

37D30 Partially hyperbolic systems and dominated splittings
37C40 Smooth ergodic theory, invariant measures for smooth dynamical systems

Keywords:

partially hyperbolic systems; SRB measures; smooth ergodic theory; u-Gibbs states; abelian Anosov actions; rapidly mixing diffeomorphisms

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