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The inclusion theorem for multiple summing operators. (English) Zbl 1064.47057

Stud. Math. 165, No. 3, 275-290 (2004).

Let X be a Banach space, X^* be its dual and B_X its unit ball. For a finite sequence $(x_i)_{i=1}^m \subset X$ and $1 \leq p < \infty$, write $\left\|(x_i)_{i=1}^m\right\|_p^\omega$ for

$$\sup\left\{\left(\sum_{i=1}^m |x^*(x_i)|^p\right)^{1/p} : x^* \in B_{X^*}\right\}.$$

Let $1 \leq p < \infty$. A multilinear operator $T : X_1 \times \cdots \times X_n \to Y$ is multiple p-summing if there exists a constant $K > 0$ such that for every choice sequence $(x_{i_j}^j)_{i_j=1}^{m_j} \subset X_j$,

$$\left(\sum_{i_1,\cdots,i_n=1}^{m_1,\cdots,m_n} \left\|T(x_{i_1}^1,\ldots,x_{i_n}^n)\right\|^p\right)^{1/p} \leq K \prod_{j=1}^n \left\|(x_{i_j}^j)_{i_j=1}^{m_j}\right\|_p^\omega.$$

In that case, the multiple p-summing norm $\Pi_p(T)$ of T is defined the minimum K such that the above inequality holds. Denote by $\Pi_p^n(X_1,\ldots,X_n;Y)$ the class of multiple p-summing n-linear operators, which is a Banach space with the norm Π_p. A Banach space X is called to be a GT space, i.e., X satisfies Grothendieck's theorem, if there exists $K > 0$ such that each linear operator $u : X \to \ell_2$ is 1-summing and satisfies $\Pi_1(u) \leq K\|u\|$. In this paper, the author proves that for $1 \leq p \leq q < 2$, each multiple p-summing multilinear operator between Banach spaces is also q-summing. The author also gives an improvement of this result for the case of an image space of cotype 2.

A multilinear operator $T : H_1 \times \cdots \times H_n \to H$ between Hilbert spaces is said to be Hilbert-Schmidt if there exists $K > 0$ such that

$$\left(\sum_{i_1\in I_1,\ldots,i_n\in I_n} \|T(e_{i_1}^1,\ldots,e_{i_n}^n)\|^2\right)^{1/2} < K,$$

where $(e_{i_j}^i)_{i_j\in I_j} \subset H_j$ is an orthonormal basis, $1 \leq j \leq n$. In this case, the above least constant K is called the Hilbert-Schmidt norm of T. Denote by $S_2^n(H_1,\ldots,H_n;H)$ the class of Hilbert-Schmidt multilinear operators. In this paper, the author proves that if $H_1,\ldots,H_n$ and H are Hilbert spaces and $T : H_1 \times \cdots \times H_n \to H$ is a multilinear operator, then $T \in S_2^n(H_1,\ldots,H_n;H)$ if and only if $T \in \Pi_p^n(H_1,\ldots,H_n;H)$ for every $p \in [1,\infty)$ if and only if $T \in \Pi_p^n(H_1,\ldots,H_n;H)$ for some $p \in [1,\infty)$, which is a multilinear version of the classical characterization of Hilbert-Schmidt linear operators given by *A. Pełczyński* in [Stud. Math. 28, 355–360 (1967; Zbl 0156.38001)]. Moreover, the author proves that for GT spaces, every multilinear operator into a Hilbert space is 1-summing with an optimal constant, which is a multilinear generalization of Grothendieck's theorem for GT spaces.

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MSC:

47H60 Multilinear and polynomial operators
46B25 Classical Banach spaces in the general theory
46C99 Inner product spaces and their generalizations, Hilbert spaces

Cited in **2** Reviews
Cited in **32** Documents

Keywords:

Hilbert-Schmidt operators; p-summing operators; absolutely summing operators; multilinear operators; Grothendieck's theorem

Full Text: DOI

Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities
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