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$L_{3,\infty}$-solutions of Navier-Stokes equations and backward uniqueness. (English. Russian original)

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Let $f(x,t) \in L_s(\mathbb{R}^3)$ with respect to the variable $x \in \mathbb{R}^3$. Let $L_{s,\infty}$ denote the space with the norm $\|f\|_{s,\infty} = \operatorname{ess\,sup}_{t\in(0,T)} \|f(\cdot,t)\|_s$. The authors prove that any weak Leray-Hopf solution $v(x,t)$ of the Cauchy problem for the Navier-Stokes equations satisfying the additional condition $v \in L_{3,\infty}(Q_T)$ belongs to $L_5(Q_T)$. It is smooth and unique on $Q_T = \mathbb{R}^3 \times (0,T)$.

The authors also study the conditions when v is Hölder continuous in a ball from $\mathbb{R}^3$. The heat operator $\partial_t + \Delta$ is considered on $Q_+ = \mathbb{R}^n_+ \times (0,1)$ in a class of generalized functions. It is proved that if $|\partial_t u + \Delta u| \leq c(|\nabla u| + |u|)$ on Q_+, $u(\cdot,0) = 0$ on $\mathbb{R}^n_+$ and $u(x,t) \leq \exp(M|x|^2)$ for all $(x,t) \in Q_+$, then $u(x,t) \equiv 0$ on Q_+.

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MSC:

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76D05 Navier-Stokes equations for incompressible viscous fluids

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