Antman, Stuart S.

Nonlinear problems of elasticity. 2nd, revised and extended ed. (English) Zbl 1098.74001
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The introductory chapter 1 contains the necessary facts from mathematical analysis and used notations. Materials from linear and nonlinear functional analysis and, in particular, elements of bifurcation theory and degree theory are presented in conclusive Chs. 19–21 as appendices.

The purpose of Ch. 2 is to give the derivation of quasilinear PDE systems governing the large motion of nonlinearly elastic and viscoelastic strings. Special attention is paid to the principle of virtual power and to the equivalent impulse-momentum law as physical and mathematical generalizations of the equations of motion, playing essential roles in the treatment of initial and boundary conditions, jump conditions, variational formulations, and perturbation and approximation methods. Ch. 3 is devoted to elementary problems for inextensible strings. These are the determination of equilibrium states when the string hangs between two points and is subjected to various loads: (1) the weight of the string; (2) a vertical load of constant intensity per horizontal distance; (3) a normal pressure of constant intensity; (4) a normal pressure varying linearly with depth; (5) the attraction to a fixed point. Here the author considers also dynamic problems with solution of travelling wave and radial oscillation type and combined whirling and radial motions. In Ch. 4 equilibrium problems for the planar deformation of elastic rods are formulated and studied. These are equilibrium states of straight rods under terminal loads, equilibrium of rings under hydrostatic pressure, asymptotic shape of inflated rings, straight configurations of whirling rods and simultaneous whirling and breathing oscillations of a ring.

The main part of the book begins with Ch. 5, where on the base of Chs. 19–21 the author gives an introduction to bifurcation theory with applications to elasticity. Here, after many examples of classical bifurcational buckling problem of elasticity, the author proves local and one-parameter global bifurcation theorems in his own treatment, more convenient for elasticity problems. Further, an application of these theorems to elastica is given and perturbation methods for solutions of continuation problem are described. The chapter finishes with a review of stability results for bifurcating solutions. Ch. 6 is devoted to global bifurcation problems for strings, rods and arches (problems for whirling strings, drawing of strings, planar buckling problems for rods, buckling of arches).

Ch. 7, "Variational methods" describes some applications of the calculus of variations to one-dimensional problems of nonlinear elasticity. The main subject here is the description of methods for static problems. The author shows that the potential-energy functional can be minimized and the minimizer actually satisfies Euler-Lagrange equations. Euler-Lagrange equations are obtained for problems subjected to isoperimetric constraints and for problems for whirling rods. In Ch. 8 the author formulates the general dynamical theory of rods deforming in space and suffering flexure, torsion, extension and shear. Thus, the geometrically exact special Cosserat theory of rods is developed. Hamilton's principle for strings from Ch. 2 is generalized to hyperelastic rods moving in space under conservative loads. In Ch. 9, after summary of the governing equations for spatial deformations of nonlinearly elastic rods, the author investigates the Kirchhoff problem for helical equilibrium states and obtains general solutions of equilibria, travelling waves in straight rods, and examines the buckling in space of a transversely hemitropic rod subjected to terminal thrust and torque. The author investigates also the bifurcation problem of lateral instability, when a rod has a plane of symmetry and is subjected to a load with the same symmetry.

In Ch. 10 the geometrically exact theories are given for axisymmetric deformations of plates and shells that can suffer flexure, mid-surface extension and shear. The following problems are investigated: buckling of a transversely circular plate and trivial states of aeolotropic circular plates; buckling of spherical shells; buckling of cylindrical shells; asymptotic shape of inflated shells; membranes; everted equilibrium states.

For the further presentation the author gives a detailed introduction to tensor algebra and analysis (Ch. 11) and on this base introduces three-dimensional continuum mechanics (Ch. 12) based on Lagrangian description of motion, with modifications needed for Eulerian formulation. In Ch. 13 the governing equations of nonlinear elasticity are described with discussion of constitutive restrictions and their impli-

cations. Problems of nonlinear elasticity are described in Ch. 14: elementary static problems; torsion, extension, inflation and shear of (compressible) annular sector; torsion and related equilibrium problems for incompressible bodies; flexure, extension and shear of a (compressible) block; dilatation, cavitation, inflation and inversion and other semi-inverse problems; universal and non-universal deformations; antiplane problems. Perturbation methods are presented in application to spatial buckling and necking of an incompressible, homogeneous, isotropic elastic cylinder under prescribed end displacement, and to the instability of an annulus welded to a spinning disc. The author also studies the problems of deformation of a homogeneous, isotropic incompressible body under a constant dead normal traction, radial motions of an incompressible tube, and standing shear waves in an incompressible layer.

Large-strain plasticity problems are investigated in Ch. 15. Chs. 16 and 17 are devoted to the general theory of rods (geometry of rod-like bodies, exact equations of motion; semi-intrinsic theories; induced theories of rods; rods with two directors; necking of elastic rods with a plane of symmetry); the treatment of incompressibility and general theories of plates and shells (induced shell theories, shells with one director; drawing and twisting of elastic plates; axisymmetric motions of axisymmetric shells; global buckled states of a Cosserat plate; thickness parameter, eversion; the treatment of incompressibility; intrinsic theory of special Cosserat shells; membranes). In sections 10 and 11 of Ch. 17 the author gives asymptotic methods for von Kármán equations for plates, where the small parameter is the (half-)thickness, together with the justification of shell theories as asymptotic limits.

The conclusive Ch. 18 treats a collection of dynamical problems for elastic and viscoelastic bodies with applications of modern theories of quasilinear hyperbolic and parabolic systems to elasticity. Here the following problems are discussed: shearing motions of viscoplastic layers; dissipative mechanisms and the bounds they induce; shock structure, admissibility and travelling waves; travelling shear waves in viscoelastic media; blow-up in three-dimensional hyperelastisity.

Reviewer's summary: The second extended edition of the reviewed monograph gives a fundamental presentation of problems of nonlinear elasticity. Every chapter is equipped by instructive exercises, unsolved problems and exhaustive historical comments. The book could be very useful to applied mathematicians and engineers using in their works the elasticity theory and, in particular, to specialists dealing with applications of differential equations and bifurcation theory.

Reviewer: Boris V. Loginov (Ul'yanovsk)

MSC:

74-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to mechanics of deformable solids	Cited in **1** Review Cited in **169** Documents
74B20	Nonlinear elasticity	
74Kxx	Thin bodies, structures	
74G60	Bifurcation and buckling	

Keywords:

rod; string; bifurcation; buckling; plate; shell

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