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Singular stochastic differential equations. (English) Zbl 1071.60003

Lecture Notes in Mathematics 1858. Berlin: Springer (ISBN 3-540-24007-1/pbk). viii, 128 p. (2005).

The main aim of this outstanding research monograph on stochastic differential equations is to introduce a class of points termed isolated singular points. Stochastic differential equations possessing such points, called singular stochastic differential equations, arise often in theory and in applications. However, known conditions for the existence and the uniqueness of a solution typically fail for such equations. The book studies the existence, the uniqueness, and the qualitative behaviour of solutions of singular stochastic differential equations. This is done by providing a qualitative classification of isolated singular points into 48 possible types. The monograph also includes an overview of the basic definitions and facts related to the stochastic differential equations (different types of existence and uniqueness, martingale problems, solutions up to a random time, etc.) as well as a number of important examples.

The book is organized in five chapters and has two appendices. Chapter 1 is an overview of basic definitions and facts related to stochastic differential equations, and also includes several important examples of such equations. The relationships between different types of existence and uniqueness of weak and strong solutions are described and all the possible combinations of existence and uniqueness of solutions are characterized.

In Chapter 2, the authors introduce the notion of a singular point and study the existence, the uniqueness, and the qualitative behaviour of a solution in the right-hand neighbourhood of isolated singular points into seven possible right types.

In Chapter 3, the authors investigate the existence, the uniqueness, and the qualitative behaviour of a solution in the two-sided neighbourhood of an isolated singular point. The effects brought by the combination of right and left types are considered. Since there exist seven possible right types and seven possible left types, there are 49 feasible combinations. One of these combinations corresponds to a regular point, and therefore, an isolated singular point can have one of 48 possible types. It turns out that the isolated singular points of only four types can disturb the uniqueness of solutions. They are called branch points and characterize all the strong Markov solutions in the neighbourhood of such a point. All the results of Chapters 2 and 3 can be applied to the case of local solutions, that is, solutions up to a random time.

In Chapter 4, the behaviour of a solution “in the neighbourhood of infinity” is investigated, which leads to the classification at infinity. This classification shows that infinity can have one of three possible types and whether a solution can explode into infinity. Feller’s test for explosions turns out to be a consequence of this classification. The results obtained in the previous chapters are applied to study the existence, the uniqueness, and the qualitative behaviour of global solutions, that is, solutions in the classical sense. This is done for stochastic differential equations that have no more than one singular point.

In Chapter 5, the power equations are considered, and a simple procedure to determine the type of zero and the type of infinity for these stochastic differential equations is proposed. Moreover, the question: which type of zero and which type of infinity are possible for the stochastic differential equations with a constant-sign drift are studied. The known results from the stochastic calculus used in the proofs are contained in Appendix A, while the auxiliary lemmas are given in Appendix B. The monograph includes seven figures with simulated paths of solutions of singular stochastic differential equations.

Reviewer: [Pavel Gapeev \(Moskva\)](#)

MSC:

- 60-02 Research exposition (monographs, survey articles) pertaining to probability theory
- 60H10 Stochastic ordinary differential equations (aspects of stochastic analysis)
- 60J25 Continuous-time Markov processes on general state spaces
- 60J60 Diffusion processes
- 60G17 Sample path properties
- 60G44 Martingales with continuous parameter

Cited in **36** Documents**Keywords:**

existence and uniqueness of weak and strong solutions; (isolated) singular points; branch points; power stochastic differential equations; diffusion processes

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