Sinestrari, Carlo

Semiconcavity of the value function for exit time problems with nonsmooth target. (English)

Zbl 1064.49024
Commun. Pure Appl. Anal. 3, No. 4, 757-774 (2004).

The author proves a new semiconcavity theorem for the value function

$$V(x) := \inf_{u(.)} \int_0^{\tau(x;u(.))} L(y(t))dt,\ x \in \mathcal{R} := \mathrm{dom}(V(.))$$

of the *exit time problem* defined by the control system

$$y'(t) = f(y(t), u(t)),\ u(t) \in U,\ x(0) = x \in R^n,\ \tau(x; u(.)) := \inf\{t \geq 0;\ y(t) \in \mathcal{K}\}$$

where, in contrast with previous work on this topic, the target $\mathcal{K} \subset R^n$ is an arbitrary closed set with compact boundary while the "vectograms" $f(x, U)$ are assumed to be smooth and convex.

The main result of the paper may also be interpreted as a regularity result for the viscosity solutions of the associated HJB equation

$$H(x, DV(x)) = 0,\ x \in \mathcal{R} \setminus \mathcal{K},\ H(x, p) := \inf_{u \in U}[- < f(x, u), p > -L(x)],$$

$$V(x) = 0, x \in \partial\mathcal{K},\ \lim_{x \to \partial\mathcal{R}} V(x) = +\infty$$

and may be related to some of the previous results on this topic.

Reviewer: Ştefan Mirică (Bucureşti)

MSC:

49L20	Dynamic programming in optimal control and differential games	Cited in **1** Review
49L25	Viscosity solutions to Hamilton-Jacobi equations in optimal control and differential games	Cited in **12** Documents
35D10	Regularity of generalized solutions of PDE (MSC2000)	
26B25	Convexity of real functions of several variables, generalizations	

Keywords:

optimal control; exit time problem; dynamic programming; value function; semiconcavity; Hamilton-Jacobi equation; viscosity solutions

Full Text: DOI