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A unified approach to combinatorial formulas for Schubert polynomials. (English)

Zbl 1056.05146

J. Algebr. Comb. 20, No. 3, 263-299 (2004).

Summary: Schubert polynomials were introduced in the context of the geometry of flag varieties. This paper investigates some of the connections not yet understood between several combinatorial structures for the construction of Schubert polynomials; we also present simplifications in some of the existing approaches to this area. We designate certain line diagrams for permutations known as rc-graphs as the main structure. The other structures in the literature we study include: semistandard Young tableaux, Kohnert diagrams, and balanced labelings of the diagram of a permutation. The main tools in our investigation are certain operations on rc-graphs, which correspond to the coplactic operations on tableaux, and thus define a crystal graph structure on rc-graphs; a new definition of these operations is presented. One application of these operations is a straightforward, purely combinatorial proof of a recent formula due to *A. S. Buch, A. Kresch, H. Tamvakis, and A. Yong* [Duke Math. J. 122, 125–143 (2004; Zbl 1072.14067)], which expresses Schubert polynomials in terms of products of Schur polynomials. In spite of the fact that it refers to many objects and results related to them, the paper is mostly self-contained.

MSC:

- 05E15 Combinatorial aspects of groups and algebras (MSC2010)
- 14M15 Grassmannians, Schubert varieties, flag manifolds
- 05E10 Combinatorial aspects of representation theory

Cited in **2** Reviews
Cited in **13** Documents

Keywords:

Schubert polynomial; Young tableau; rc-graph; crystal graph; Kohnert diagram

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